Abstract

We address here the problem of estimating the time and space varying residual wavelet to be used in a broadband reflectivity inversion algorithm. The usual form of wavelet estimation uses a $L_2$ norm inversion algorithm, which have some limitations when few relevant reflection events are present within a specific time window. We propose the use of an alternative modified $L_1$ norm to obtain a more robust estimation of the residual seismic pulse. The robustness of the method is illustrated with synthetic and field data examples.

Introduction

Ideally, the ultimate goal of routine seismic processing/imaging is to obtain a set of amplitude volumes, where the amplitude of a given sample in each volume corresponds to the P wave reflection coefficient at that subsurface position for a given angle of incidence. Several steps of the seismic processing flow aim to compensate for the effects on the amplitude and phase spectra of the seismic wavelet caused during its generation, recording and propagation through the sub-surface. Ideally, these steps should lead the effective seismic pulse to approach a unit impulse.

Despite all these steps, which include debubbling, ghost removal and compensation for absorption, a residual wavelet will always be present in the final image volumes, which imposes limits for the resolution of the geological layers. One of the forms of broadening the spectrum, in order to increase the resolution of the images, is to apply a zero phase deconvolution to the seismic volumes.

Wavelet estimation is an important part of the deconvolution process (Portniaguine and Castagna, 2005), and has been the subject of recent research related to the improvement of seismic resolution (Zhou et al., 2016).

Our goal is to estimate the time and space varying residual zero-phase wavelet to be used in a sparse-oriented deconvolution algorithm used in Petrobras (Rosa, 2010). The standard form of wavelet estimation uses a $L_2$ norm inversion algorithm, which have some limitations when few relevant reflection events are present within a specific time window. To illustrate these limitations, we use as a reference throughout the paper the synthetic data corresponding to a wedge model, as shown in Figure 1-a.

Figure 1: (a) Synthetic data for a wedge model; (b) amplitude spectra for the traces in (a). Trace 15, corresponds to a single interface (with less impedance contrast) while trace 20 shows a tuning effect associated with the two reflections.
Figure 1-b presents the spectra of the traces from 1-a. The spectrum of each individual trace shows a particular pattern of resonant frequencies, which relates to the time delay between the top and bottom reflections of the wedge. An important property represented in the figure is that all the individual spectra share the same envelope, which is the spectrum of the original pulse.

As we will show in the next sessions, a modified $L_1$ norm is more suitable for a robust estimation of the residual seismic pulse, than the usual $L_2$ norm.

$L_2$ norm wavelet estimation

The first step involves the transformation of the spectrum $S(\omega)$ in the function $Y(\omega)$, which in the discrete formulation is given by

$$ Y_j = \ln \frac{S_j}{\omega_j^\alpha}, $$

(1)

where $S_j$ corresponds to the amplitude spectrum, $\omega_j$, is the frequency, and $\alpha$ is a positive exponent smaller than one.

An usual way to formulate the problem of wavelet estimation for a time window with transformed spectrum defined by $Y_j$ is to obtain the coefficients $a_i$ that defines the polynomial $P_j$ of degree $N$ (Rosa and Ulrych, 1991):

$$ P_j = \sum_{i=1}^{N} a_i \omega_j^i $$

(2)

which minimizes the $L_2$ norm error function

$$ E_2 = \sum_{j=1}^{M} |Y_j - P_j|^2. $$

(3)

The solution is given by solving for the coefficients $(a_1, a_2, \ldots, a_N)$, the set of linear equations

$$ \frac{\partial E_2(a_1, a_2, \ldots, a_N)}{\partial a_k} = 0, \quad \text{for} \quad k \in \{1, N\} $$

(4)

which leads to

$$ \sum_i \sum_j \omega_j^{i+k} a_i = \sum_j Y_j \omega_j^k $$

(5)

The wavelets estimated with the $L_2$ norm (Equation 5), for 4 selected traces from Figure 1-a, are presented in Figure 2. The limitations of the $L_2$ solution to retrieve the true wavelet become apparent from this figure.

Modified $L_1$ norm wavelet estimation

In order to improve the pulse estimation, let’s first analyse the residuals represented in Equation 3. Figure 3-a shows, for trace 50, the transformed spectrum $Y_j$, and the polynomial solution $P_j$, and 3-b the $L_2$ residuals before squaring (that is, the $L_1$ residuals) for the solution. In order to achieve the goal of the solution $P_j$ to approach the envelope of $Y_j$ we should have an error function that penalizes only the positive part of $Y_j - P_j$ in Figure 3-b.

Following this reasoning, we present Equation 6 as a first attempt to define an error function whose minimization would lead to the envelope of the trace spectrum as the solution.

$$ E_1^+ = \sum_{j=1}^{M} \{(Y_j - P_j) + |Y_j - P_j|\}. $$

(6)

Figure 2: $L_2$ norm estimation results for 4 traces from Figure 1. The traces are: 30 (top left), 35 (top right), 50 (bottom left), 60 (bottom right).

As we will discuss later in this paper, the solution of this equation is achieved by an iterative inversion algorithm. The residuals represented in Equation 6, for
the specific case of the $L_2$ solution for trace 50 are depicted in Figure 4.

![Figure 3](image)

Figure 3: (a) Transformed spectrum $Y_j$ and the $L_2$ norm solution for the fitting polynomial $P_j$; (b) Residuals for the $L_2$ norm solution.

Although the solution of Equation 6 converges to the desired envelope for low order polynomials (up to 4th order), it becomes unstable for higher order, as illustrated in Figure 5. The evolution with iteration of the residuals are shown in 5-a, while the evolution of the solution is shown in 5-b. Although the first iterations show convergence to the desired solution, the next iterations diverge. The reason is that the error function, as defined by Equation 6, becomes zero for all polynomial functions that have no intersection with the transformed spectrum. The envelope is just one of these functions.

In order to avoid solutions that diverge from the desired envelope, we must introduce another term in the objective function, that penalizes the negative residuals, but in a smaller scale than the positive residuals. We define thus a new error function:

$$E_{1+}^\pm = \sum_{j=1}^{M} \left\{ \left[ (Y_j - P_j) + |Y_j - P_j| \right] - \beta \left[ (Y_j - P_j) - |Y_j - P_j| \right] \right\},$$

![Figure 4](image)

Figure 4: Residuals for the objective function defined by equation 6, for the same functions $Y_j$ and $P_j$ used in Figure 3.

![Figure 5](image)

Figure 5: (a) Evolution with iteration, of the residuals for the $E_{1+}$ solution; (b) evolution with iteration of the fitting function $P_j$ for the $E_{1+}$ solution.
where $\beta$ is a scalar smaller than one. After regrouping the terms

$$E_i^{\pm} = \sum_{j=1}^{M} \left[ (1-\beta)(Y_j - P_j) + (1+\beta)|Y_j - P_j| \right]. \quad (7)$$

The residuals represented in Equation 7, for the specific case of the $L_2$ solution for trace 50 are depicted in Figure 6. If we compare with Figure 4, we can see that the negative residuals are now present with a small weight.

In Equation 8 $\beta^m$ is an iteration dependent coefficient, that decreases at a specified rate as the iterations proceed. The initial value must be smaller than one.

Figure 7 shows the evolution of the residuals, as well as the evolution of the solution for a polynomial of $6^{th}$ order. The wavelets estimated with

the modified $L_1$ norm (using the iterative solution of Equation 8) for the same 4 traces used in Figure 2 are presented in Figure 8. The wavelets estimated with the modified $L_1$ norm are clearly superior to retrieve the true seismic pulse when compared to the ones estimated with the $L_2$ norm.

**Examples**

The first example refers to the same wedge model, used throughout the paper to illustrate the concepts that support the proposed form for the modified $L_1$ norm. We applied the wavelet estimation inversion for all traces, followed by the reflectivity inversion.
The results are presented in Figure 9. The retrieved reflectivity using the $L_1$ norm wavelets are less contaminated by artifacts when compared to the reflectivity obtained with the $L_2$ norm wavelets. The differences between the $L_2$ and the modified $L_1$ norm estimates become more clear when we compare the integrated reflectivities, which correspond to a band-limited version of the acoustic impedance. Figure 10 shows the impedances from the two reflectivity panels of Figure 9 (one for each norm), as well as the impedance from the reflectivity inversion using the true wavelet. The modified $L_1$ result is almost identical to the one with the correct wavelet.

The second example involves the application of the modified $L_1$ norm to a field data from Santos basin (Brazil). A reflectivity inversion process was applied to generate band-limited impedance data. Figure 11-a shows the amplitude data, while 11-b and 11-c, show the band-limited impedance using the $L_2$ and $L_1$ norm respectively. The increase in resolution with the $L_1$ norm is clear from the figure.

Although no spectral balancing algorithm have been applied in the process, the improvement achieved with the $L_1$ norm can be related to the better spectral homogeneity present in it’s inverted reflectivity, as demonstrated by Figure 12.

Conclusions

Wavelet estimation plays an important role in the process of reflectivity inversion from band-limited seismic data. The process, considering zero-phase wavelets, is usually based on a smoothed version of the spectrum of the seismic trace. This process may be carried on a time and space window basis.

We introduced a modified $L_1$ norm, which focus on the minimization of positive residuals and leads to a stable estimation of the spectrum envelope. Synthetic and real data examples show that this new norm provide a superior estimation of the residual seismic pulse when compared to the wavelets estimated using the $L_2$ norm.

Figure 9: (a) Synthetic wedge model data; (b) sparse deconvolution with $L_2$ norm wavelet estimation; (c) sparse deconvolution with the modified $L_1$ norm wavelet estimation.
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References


Figure 10: Retrieved impedance from deconvolution with: (a) the true wavelet; (b) the $L_2$ norm wavelet; (c) the $L_1$ norm wavelet.

Figure 11: Real data example: (a) amplitude data; (b) band-limited impedance with $L_2$ norm wavelet; (c) band-limited impedance with $L_1$ norm wavelet.

Figure 12: Spectra from Figure 11: amplitude data (black); reflectivity with $L_2$ norm wavelet (blue); reflectivity with $L_1$ norm wavelet (red).