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A MATHEMATICAL MODEL FOR WIRELINE LOGS FILTERING USING DIFFERENTIAL FUZZY PARAMETERS

Carlos Eduardo Guerra[1], André José Neves Andrade[2] and Ivnon José de Farias Lima[2]

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ABSTRACT. Signal processing is a set of techniques used to extract data from any signal. A signal may be mathematically characterized as a function of several variables (parameters), such as time, distance, resistivity, or radiance. A signal is usually acquired using one or more analog or digital devices, such as a temperature sensor, a digital camera, or a resistivity probe. After acquisition, the processing depends on the nature of the signal and its information. In addition to the information being studied, the recorded signal may contain noise, which hinders the extraction of information, leading to ambiguous or erroneous results. The objective of this study is to present a non-linear technique for noise attenuation in well logs using fuzzy sets and fuzzy logic. The proposed filter evaluates the continuity of the log measurements through the use of two differential parameters. Sudden leaps in the measurements may indicate the presence of noise; therefore, for each point in the log, the filter evaluates the degree of discontinuity and provides a correction value to be applied. Differential fuzzy filtering is applied to data from synthetic and real well logs to conduct a performance evaluation using the MSE (mean square error).

Keywords: well logging, noise, digital filtering and fuzzy systems.

RESUMO. Podemos caracterizar o processamento de sinais como um conjunto de técnicas utilizadas para extrair informações de um sinal qualquer. Um sinal pode ser caracterizado matematicamente como sendo uma função de várias variáveis (parâmetros), como exemplo, tempo, distância, resistividade, radiância, por exemplo. A aquisição de um sinal é normalmente feita por um ou mais dispositivos analógicos e/ou digitais, como exemplos, um sensor de temperatura, uma câmera digital, uma sonda de resistividade, dentre outros. O tipo de processamento a ser aplicado, após aquisição, dependerá da natureza do sinal e da sua informação. O sinal registrado poderá conter, além da informação desejada, também uma quantidade de informaçõessem nexo ou interesse, denominadas por ruído. A presença de ruídos nos sinais pode prejudicar os processos de extração da informação desejada, levando a resultados ambíguos e sem nexo. Assim o objetivo deste trabalho é mostras uma técnica não linear para a remoção de ruídos em perfis de poços utilizando conjuntos e lógica fuzzy. O filtro proposto avalia a continuidade das medidas do perfil através da utilização de dois parâmetros diferenciais. Saltos abruptos nas medidas podem indicar a presença de ruídos, assim, para cada ponto no perfil, o filtro avaliará o grau de descontinuidade e atribuirá um valor de correção a ser aplicado. Finalmente, a filtragem fuzzy diferencial será aplicada em dados de perfis de poços sintéticos e reais, onde procederemos a uma avaliação de desempenho utilizando o parâmetro MSE (erro médio quadrático).

Palavras-chave: perfis de poços, ruídos, filtragem digital e sistemas fuzzy.

[1]UFOPA – Universidade Federal do Oeste do Pará, Campus Universitário da UFOPA, Avenida Marechal Rondon, s/n, Caranazal, 68040-070 Santarém, PA, Brasil. Phone: +55(93) 3522-4404 – E-mails: guerra@ufpa.br; carlos.guerra@pq.cnpq.br

[2]UFPA/CPGF, UFPA – Universidade Federal do Pará, Programa de Pós-Graduação em Geofísica, Rua Augusto Correa, 1, Campus Básico, Guamá, Caixa Postal 8608, 66075-110 Belém, PA, Brasil. Phone: +55(91) 3201-7690 – E-mails: aandrade@ufpa.br; ivnonfisica@hotmail.com

INTRODUCTION

Signal processing consists of the analysis or modification of signals to extract information or format them for a particular application. This processing may be analog or digital. Signal processing techniques can be very useful in the control and analysis of physical systems in various fields, such as engineering, economics, biology, geosciences, and health (Sonka et al., 1993; Steven, 1999). Filtering is a common signal processing technique that is often used to remove noise to improve the quality of interpretation and the use of data. In this work, we focus on digital filtering applied to well logs, particularly algorithms based on fuzzy logic and fuzzy sets (Zadeh, 1965).

A well log is a representation of the variations of a physical properties measured along the depth of a borehole. In the past, analog measurements were taken and the resulting log was shown graphically on paper. Measurements are now digitally recorded and stored in ASCII-type files, as standardized by API[1] (Serra, 1984; Serra & Andreani, 1991). These measurements are usually contaminated by noise due to issues with sonde calibration, mechanical problems, and borehole interference. To minimize the effects of noise in the well logs interpretation, techniques based on Fourier analysis, inversion procedures (Claerbout, 1985; Guerra, 1994) and non-linear techniques (Anderson, 2001) can be used. For example, Jesus et al. (2003) introduced a method of data filtering to better delineate lithologic data from wells. This method is based on the Lp norm and represents a generalization of the concept of the mean. However, in many cases technological limitations (e.g., the tool response functions) and numerical problems (e.g., the transfer functions for the filters, computational errors, signal-to-noise ratio estimates) cannot always be circumvented in a simple and satisfactory way. Thus, cognitive algorithms, including fuzzy systems (Zadeh, 1965; Takagi & Sugeno, 1985), offer new alternatives for processing of data involving uncertainties and ambiguities (e.g., well logs). Some authors use fuzzy logic to filter image data, including Kaoru Arakawa (1996), who proposed a filter to eliminate impulse noise in images. The filter is obtained through a weighted sum of input signals and the output of the median filter; the weight is defined on the basis of fuzzy rules regarding the states of the input signal. Morillas et al. (2008) has also explored this technique and described a new filter that uses a reduced ordering of color vectors to detect and eliminate impulse-type forms. The purpose of this filter is to use adaptive statistics to simultaneously remove impulses and preserve the image, edges, and details.

Fabrizio Russo & Giovanni Ramponi (1996) presented a new operator that adapts the fuzzy logic approach to improve images corrupted by impulse noise. The proposed operator is based on two phases and is able to remove very strong noise while simultaneously preserving the image details. The first phase (the action detection module) aims to detect noise pulses by considering the differences among neighboring pixels, allowing the selection of a possible correction. The second phase (the action adaptation module) modifies the correction value to improve the preservation of the details.

Following the same reasoning, Schulte et al. (2006) described a new algorithm, FIDRM[2], that can be applied to images with a mixture of impulse noise and other types of noise. The result is an image with very little impulse noise. This non-linear filtering technique contains two stages: the first consists of detecting and reducing impulse noise, and the second involves the preservation of the sharpness of the image edges. Based on the concept of a fuzzy set, the method uses a fuzzy gradient to detect impulse noise.

Nejad et al. (2008) presented a new fuzzy filter to reducenoise in images containing constructive noise. This filter also consists of two phases. The first involves the processing of all image pixels to determine the noisy pixels using a fuzzy system that associates a degree to each pixel (i.e., a real number in the interval $[0, 1]$), which indicates the probability that this pixel is not noisy. In the second stage, a new fuzzy rule uses the output of the previous stage to weight the contributions from neighboring pixels.

Many studies related to filtering use neighboring data to analyze a given point. This strategy is based on the concept of function continuity, where the smooth behavior is expected near of the each point.

METHODOLOGY

Initial considerations

Fuzzy systems are based on operations that involve fuzzy sets and logic (Aguiar & Oliveira, 1999). Fuzzy logic differs from classical logic in the allocation of "true" and "false" values. In classical theory, an element belongs to a set if and only if it has a value 0 or 1. A value of 0 indicates complete exclusion, and a value of 1 indicates complete inclusion. Therefore, given a universe U and

[1] API is the acronym for American Petroleum Institute.

[2] FIDRM is the acronym for "Fuzzy Impulse noise Detection and Reduction Method".

any element $x \in U$, the membership function $\mu_A(x)$ in relation to a set $A \subseteq U$ is given by Barron (1993):

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A\,. \end{cases} \tag{1}$$

Different from classical theory, a fuzzy set A defined in the universe X is characterized by a membership function μ_A, which relates the elements of X in the interval $[0, 1]$:

$$\mu_A(x) \to [0, 1]\,. \tag{2}$$

Thus, the membership function associates each element x belonging to X with a real number $\mu_A(x)$ in the interval $[0, 1]$, which represents the degree of membership of the element x to the set A, i.e., how much the element x belongs to set A:

$$\begin{cases} \mu_A(x) : X \to [0, 1], \\ \mu_A(x) = 0, \ \text{if } x \notin A, \\ 0 < \mu_A(x) < 1, \ \text{(parcial membership in } A\text{)}, \\ \mu_A(x) = 1, \ \text{(total membership in } A\text{)}, \\ A = \left\{(x, \mu_A(x)) \big/ x \in X\right\}. \end{cases} \tag{3}$$

Fuzzy logic can be seen as a generalization of classical logic, where any value in the interval $[0, 1]$ can be assumed.

The method

Fuzzy systems are used with data that carry a certain degree of uncertainty, inaccuracy, or ambiguity. The presence of noise in the wireline measurements further increases these uncertainties. Given the previously mentioned characteristics, we developed a mathematical model, based on fuzzy logic, for the removal of noise from well logs. For each point of the well log, differentials to the left and to the right are calculated. The signal continuity is then evaluated by testing the differential signals through inference rules. Differentials with opposite sign indicate continuity in the log, while those ones with the same sign indicate a leap at the point considered. As a result of the defuzzification process, a correction factor is generated to eliminate or reduce this leap. Figure 1 shows the simplified block diagram of this application of a differential fuzzy filter.

In the diagram (Fig. 1), fr is the well log (noise and information), Cor are the correction factors calculated by the filter, D^e and D^d are the differential parameters in the first iteration, D_n^e and D_n^d are the differential parameters in the n-iteration, $\tilde{f}_n$ is the filtered well log in the n-iteration, and $\tilde{f}$ is the filtered well log after N iterations. The mathematical model of the fuzzy system proposed in this study consists of the following steps:

Data entry

Differentials of well log values are adopted as input variables, which are defined as follows:

Def. 1: The differential parameters to the left, $D_i^e(z_i)$ and $Dn_i^e(z_i)$, and to the right, $D_i^d(z_i)$ and $Dn_i^d(z_i)$,for each depth point z_i are extracted from the well log fr (the raw wireline log) and $\tilde{f}_n$ (the filtered log in the n-th iteration)and are defined as:

- Initial differential parameters (first iteration):

$$\begin{cases} D_i^e(z_i) \equiv fr_{i-1}(z_{i-1}) - fr_i(z_i) \\ D_i^d(z_i) \equiv fr_{i+1}(z_{i+1}) - fr_i(z_i) \end{cases} \tag{4}$$

- Initial differential parameters in the n-th iteration:

$$\begin{cases} Dn_i^e(z_i) \equiv \tilde{f}_{n,i-1}(zi-1) - \tilde{f}_{n,i}(z_i) \\ Dn_i^d(z_i) \equiv \tilde{f}_{n,i+1}(zi+1) - \tilde{f}_{n,i}(z_i) \end{cases} \tag{5}$$

Parameter initialization for the fuzzy filter

In this step, the filter is initialized using the initial estimate of the wireline log noise level (universe of discourse) and the number of iterations that are required for filter the data. The universe of discourse is defined as:

$$Xn = [-nr, nr], \tag{6}$$

where nr is an estimate of the maximum noise level and is given as:

$$nr = \max\left(\max(|D^e|),\ \max(|D^d|)\right) \tag{7}$$

Fuzzification of the input parameters

Mapping of the input dataset is performed by calculating differential parameters to each side in one or more sets in the interval $[0, 1]$. This step also includes the definition of the membership functions for the variables. In the membership functions, we use the Gaussian function (Gomide et al., 1994) given by:

$$gaussmf(x, c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}, \tag{8}$$

where c is the mean and σ is the standard deviation. From this membership function, the differential fuzzy sets to the left and right are defined as:

$$\begin{cases} FD^{TI} = \left\{\left(x, \mu_{FD^{TI}}(x)\right)\right\}, \ x \in X_n, \\ \mu_{FD^{TI}} : X_n \to [0, 1], \ \text{with} \\ \qquad \mu_{FD^{TI}}(x) = e^{\frac{-(x-c_{TI})^2}{2\sigma_{TI}^2}}, \\ TI = \{e^-, e^+, d^-, d^+\}. \end{cases} \tag{9}$$

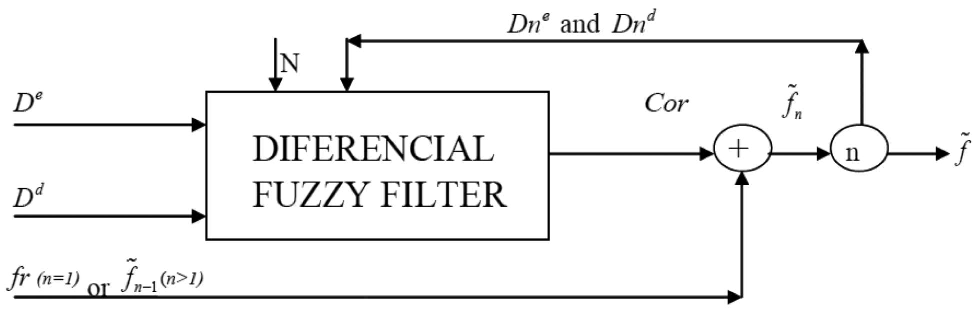

Figure 1 – Diagram of a fuzzy differential filter.

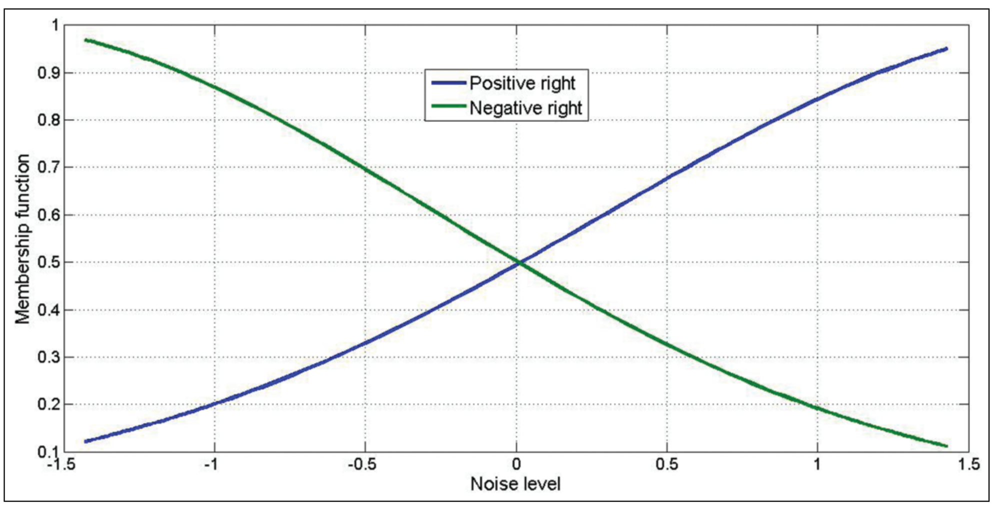

Figure 2 – Membership functions for mapping differential parameters. Blue indicates positive, and green indicates negative.

In system (9), FD^{TI} is a family of differential fuzzy sets whose superscripts TI indicate the nature of each set: Differential to the left and negative or positive, or differential to the right and negative or positive. $\mu_{FD}TI$ are the Gaussian membership functions, with c_{TI} and σ_{TI} representing the mean and the standard deviation of the measurements within the universe of discourse, respectively.

Figure 2 shows an example of how the membership functions are projected for the differential parameters to the right, either positive or negative. The parameters c_{TI} and σ_{TI} shape the Gaussian curves, defining the value of the differential parameter.

Fuzzy inference process

The fuzzy inference process consists of a series of elementary binary logic operations. The following symbols are adopted for the elementary fuzzy operators:

Def. 2: The symbols are of the following type:

- $\neg$ For the negative operator (not);
- $\bigwedge$ For the conjunction operator (and);
- $\bigvee$ For the disjunction operator (or);
- $\Rightarrow$ For proposition of the type if-then;
- $\Leftrightarrow$ For proposition of if-and-only-if;
- $\subseteq$ For inclusion.

The inference process adopted for the proposed filter consists of two steps: the evaluation of the premise of each rule (conjunction) through the T-norm operators (minimum) and the aggregation step, in which the different conclusions of the active rules under the S-norm operator (maximum) are considered and are defined as follows:

Def. 3: Norm T or T-norm – Let a, b, c and d be fuzzy sets. The T-norm operator is defined as an implementation of the binary **intersection** or operator **and**. Therefore, an operation $t : [0, 1]^2 \rightarrow [0, 1]$ satisfies:

2.1 Commutativity: $a\,\mathbf{t}\,b = b\,\mathbf{t}\,a$.

2.2 Associativity: $a\,\mathbf{t}\,(b\,\mathbf{t}\,c) = (a\,\mathbf{t}\,b)\,\mathbf{t}\,c$.

2.3 Monotonicity: if $a \le b$ and $c \le d$, then $a\,\mathbf{t}\,c \le b\,\mathbf{t}\,d$.

2.4 Limit conditions: $0\,\mathbf{t}\,a = 0$, $1\,\mathbf{t}\,a = a$.

Def. 4: Co-T-norm or S-norm – Let a, b, c and d be fuzzy sets. The S-norm operator is defined as an implementation of the binary **union** or the operator **or**. Therefore, an operation $s : [0, 1]^2 \to [0, 1]$ satisfies the following properties:

3.1 Commutativity: $a\,\mathbf{s}\,b = b\,\mathbf{s}\,a$.

3.2 Associativity: $a\,\mathbf{s}\,(b\,\mathbf{s}\,c) = (a\,\mathbf{s}\,b)\,\mathbf{s}\,c$.

3.3 Monotonicity: if $a \le b$ and $c \le d$, then $a\,\mathbf{s}\,c \le b\,\mathbf{s}\,d$.

3.4 Limit conditions: $a\,\mathbf{s}\,0 = a$, $a\,\mathbf{s}\,1 = 1$.

The properties 2.4 and 3.4 indicate that the operators T-norm and S-norm are maximum and minimum operators, respectively. The operators T-norm and S-norm are triangular norms and co-norms that are used for the operations of intersection and union in fuzzy sets formed in the previous step (fuzzification).

The assessment of the type of correction to be implemented (positive or negative) by the filter is achieved through the application of two inference rules. In this work is adopted a Mamdani-type inference system (Mamdani, 1976), which is defined by the fuzzy Cartesian product and by the fuzzy relation concept. Definitions 5, 6, and 7 below describe the Mamdani implication rule concept.

Def. 5: Let a and b be two fuzzy sets; the Cartesian product is defined as the set:

$$a \times b = \left\{ (a_i, b_j) \big/ a_i \in a \text{ and } b_i \in b \right\}.$$

Def. 6: Let a and b be two fuzzy sets; a fuzzy relation is defined as the set:

$$R_{a\times b} = \Big\{ \big((a_i, b_j),\ \mu_{a\times b}(a_i, b_j)\big) \Big/ (a_i, b_j) \in a \times b, a_i \in a, b_i \in b \quad \text{and} \quad \mu_{a\times b}(a_i, b_j) \in [0, 1] \Big\}$$

Def. 7: The Mamdani implication rule (Mamdani & Assilan, 1975; Mamdani, 1976) is applied to the outer product $a \otimes \min b$ or $\otimes \min(a, b)$, where a and b are fuzzy sets (membership functions) with universes of discourse that are not necessarily equal and $\otimes \min$ indicates the application of the operator $\min$ to each element of the Cartesian product $a \times b$. By definition:

$$a \Rightarrow b \equiv \otimes \min(a, b), \text{ namely, } \otimes \min(a, b) \equiv \left\{ \min(a_i, b_j) \big/ a_i \in a, b_j \in b \right\}.$$

The inference rules herein proposed can be expressed in words:

- $\mathbf{R_1}$: If $D_i^e(z_i)$ **and** $D_i^d(z_i)$ are positive, then the correction value Cor_i is positive.
- $\mathbf{R_2}$: If $D_i^e(z_i)$ **and** $D_i^d(z_i)$ are negative, then the correction value Cor_i is negative.

To mathematically express the rules $\mathbf{R_1}$ and $\mathbf{R_2}$, the Definitions 1 to 7 are applied, achieving:

$$\begin{cases} R = \bigvee_{k=1}^{2} \mathbf{R}_k, \\ \mathbf{R_1} : a^+ \wedge b^+ \Rightarrow Cor^+ \equiv \otimes \min\left(a^+ \wedge b^+, Cor^+\right), \\ a^+ \wedge b^+ \equiv \min\left(\mu_{FD^{e+}}(D^e), \mu_{FD^{d+}}(D^d)\right), \\ Cor^+ = \mu_{Cor^+}(COR), \\ \mathbf{R_2} : a^- \wedge b^- \Rightarrow Cor^- \equiv \otimes \min\left(a^- \wedge b^-, Cor^-\right), \\ a^- \wedge b^- \equiv \min\left(\mu_{FD^{e-}}(D^e), \mu_{FD^{e-}}(D^d)\right), \\ Cor^- = \mu_{Cor^-}(COR). \end{cases} \tag{10}$$

The terms $\otimes \min(a^- \wedge b^-, Cor^-)$ and $\otimes \min(a^+ \wedge b^+, Cor^+)$ are the Mandami implication rules, which are the set of membership functions formed by $\{\min(a^- \wedge b^-, Cor^-)\}$ and $\{\min(a^+ \wedge b^+, Cor^+)\}$, where i represents the i-th depth point considered. By analyzing each element, i.e., for each depth z_i, the operations defined in the inference system given in Eq. (10) are performed based on the **T-norm** (Figs. 3 and 4):

$$\begin{aligned} &\mathbf{R}_1 : \mu_{FD^{e+}}(D_i^e) \wedge \mu_{FD^{d+}}(D_i^d) \Rightarrow \\ &Cor^+ \equiv \min\left(\min(\mu_{FD^{e+}}(D_i^e), \mu_{FD^{e+}}(D_i^d)), Cor^+\right), \\ &Cor^+ = \mu_{COR^+}(COR) \end{aligned} \tag{11}$$

$$\begin{aligned} &\mathbf{R}_2 : \mu_{FD^{e-}}(D_i^e) \wedge \mu_{FD^{d-}}(D_i^d) \Rightarrow \\ &Cor^- \equiv \min\left(\min(\mu_{FD^{e-}}(D_i^e), \mu_{FD^{e-}}(D_i^d)), Cor^-\right), \\ &Cor^- = \mu_{COR^-}(COR) \end{aligned} \tag{12}$$

Finally, the result of the inference process is a sequence of fuzzy sets related to the application of all of the rules. To proceed to the defuzzification step, these sets must be aggregated into a single set by using an aggregation operator involving the **S-norm** (see Definition 4 and Fig. 5):

$$h_i(COR) = \max(Cor_i^+, Cor_i^-), \tag{13}$$

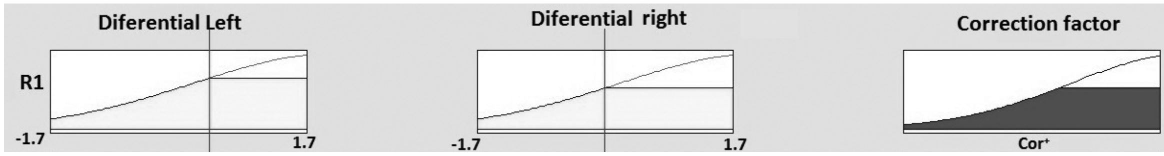

Figure 3 – Rule (1) of the Mamdani implication (T-norm or minimum operator).

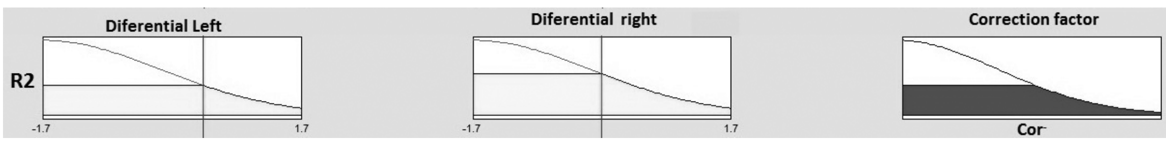

Figure 4 – Rule (2) of the Mamdani implication (T-norm or minimum operator).

where i represents each sample considered, $h_i(COR)$ is a membership function, i.e., $h_i : X_n \to [0, 1]$, and represents the aggregation curve of the rules (Fig. 5). Given the universe of discourse X_n (correction values), the corrective fuzzy output sets are defined as:

$$\begin{cases} Cor^{TI} = \{(COR, \mu_{COR^{TI}}(COR))\}, COR \in X_n, \\ e\mu_{COR^{TI}} : X_n \to [0,1] \quad \text{with} \\ \qquad \mu_{COR^{TI}}(COR) = e^{\frac{-(COR-c_{TI})^2}{2\sigma_{TI}^2}}, \\ TI = \{-, +\}, \end{cases} \tag{14}$$

where COR^{TI} is a family of corrective fuzzy sets and the superscripts TI indicate the nature (destructive of constructive correction) of each set. $\mu_{COR^{TI}}(COR)$ are Gaussian membership functions, with c_{TI} and σ_{TI} representing the mean and standard deviation of the measurements within the universe of discourse, respectively.

Defuzzification of the correction factor

The defuzzification process occurs after the inference rules are aggregated. In this stage, the correction factor is generated; it is positive if the noise is destructive and negative if the noise is constructive. The membership function values $h_j(COR)$ are used as input to the defuzzification process. The defuzzification uses the centroid method, where n is the number of points in $h_j(COR)$:

$$Cor_j = \frac{\sum_{i=1}^{n} COR_i h_j(COR_i)}{\sum_{i=1}^{n} h_j(COR_i)}. \tag{15}$$

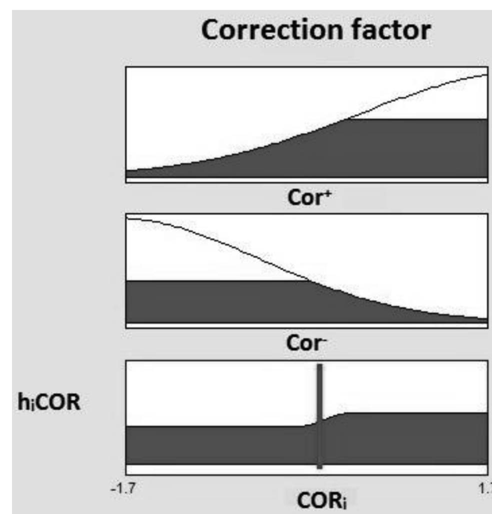

Figure 5 – The process of the aggregation of correction factors using the aggregation operator, S-norm (maximum operator).

In Eq. (15), Cor_j represents the correction to be added in the j-th measurement of the wireline log at depth z_j. The universe of discourse of the output fuzzy sets is the same as for the differential fuzzy sets because the correction values are within the estimated noise levels (Fig. 6). In Figure 6, the value in red represents the calculated centroid for the aggregation curve given.

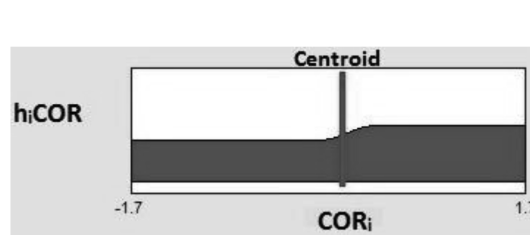

Figure 6 – The defuzzification process.

Figure 7 shows the fuzzy inference system applied according the rules and the defuzzification process.

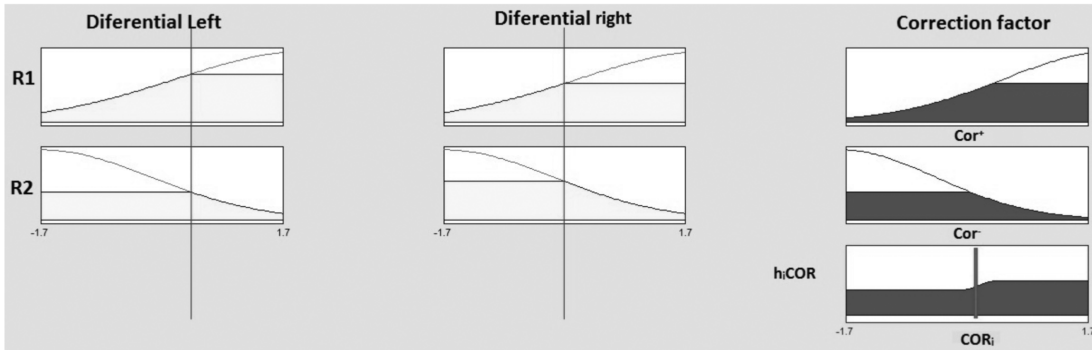

Figure 7 – The fuzzy inference system and defuzzification.

Well log correction

After defuzzification, the correction value sharpens and is applied to the well log to remove or attenuate the noise, thus improving the data quality and interpretation. The correction factor maybe constructive or destructive, depending on the effect of noise on the wireline log. The well log value is corrected by adding the value obtained during the defuzzification, i.e., for each n-th iteration:

$$\begin{cases} \tilde{f}_n = f_{n-1} + Cor, n > 1; \\ \tilde{f}_n = fr + Cor, n = 1. \end{cases} \tag{16}$$

Thus, for each measurement fr_i or $f_{n-1,i}$ at depth z_j:

$$\begin{cases} \tilde{f}_{n,i}(z_i) = fr_i(z_i) + Cor_i(z_i), n = 1, \\ \tilde{f}_{n,i}(z_i) = f_{n-1,i}(z_i) + Cor_i(z_i), n > 1. \end{cases} \tag{17}$$

Calculation of new filter input parameters for the next iteration

In this stage, the filtered well log is evaluated for the applied correction levels. This procedure is necessary for the recalibration of the fuzzy system and to perform the next iteration. For subsequent iterations, the universe of discourse and membership functions are updated to produce a lower filtering level, based on a new estimate using $\tilde{f}n$ and nr:

$$nr = \max\left(\max(Dn^{e+}), \max(Dn^{d+})\right) \tag{18}$$

The process is repeated until the last iteration, which gives the final result of the filtering process:

$$\tilde{f} = \tilde{f}_n, n = N. \tag{19}$$

As can be observed, the mathematical model of this filter is extremely simple and is based on the assumption that the noise can be constructive (increasing the signal level) or destructive (decreasing the signal level). Thus, the noise can be seen as discontinuities in the signal. The differential input parameters work as a "detector" for these discontinuities and can be used to estimate the noise level. This filter establishes the filtering level through the iterations.

RESULTS AND DISCUSSION

The filtering process was applied in synthetic and real well logs to remove the noise from these signals. For the synthetic data, induction well logs were generated based on the Doll's geometric factor (Doll, 1949; Serra, 1984; Ellis, 1987) by simulating a resistivity induction tool with two coils separated by a distance L of 1 m. The sampling interval is 0.2 m, and the initial depth z_i of 100 m. For the real data, well logs provided by PETROBRAS (Namorado Field) were used, along with core analysis and facies description. Selected interval of the gamma ray log (GR) and deep induction resistivity log (ILD) were considered. In all well logs, the differential fuzzy filter was applied with a maximum of three iterations. For comparison, we also used a classic median filter with two points. After the test with synthetic and real data, the performance of the filter was evaluated and was compared to other filtering systems.

Application to synthetic data

In the conductivity distribution model, we adopted a segmented continuous function, where the domain represents the depths and the image intervals represent the layers and their respective conductivity values. The conductivity log was generated using the Doll's geometric factor (Doll, 1949; Andrade 1992; Guerra, 1994). The resulting deep induction log was contaminated by Gaussian noise, which shows both positive and negative noise highlighted by differential parameters to each side (Fig. 8). Figure 9 shows the deep induction log filtered using the proposed fuzzy system. To demonstrate the reliability of the fuzzy filtering method, we compare it with the classical median filter method, which is widely used in signal processing (Russo & Ramponi, 1996).

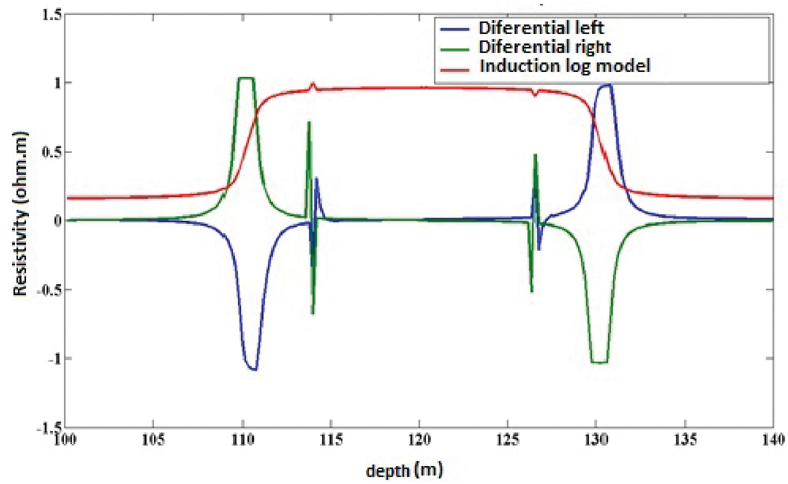

Figure 8 – The normalized model with noise highlighted by differential parameters.

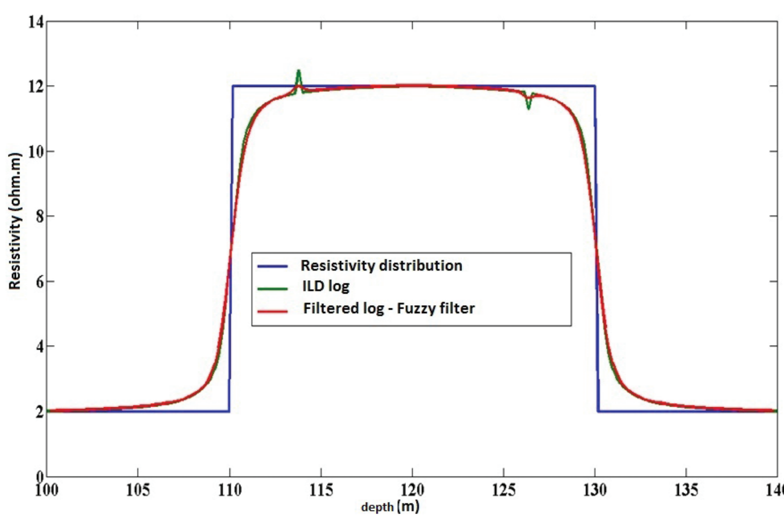

Figure 9 – The synthetic log corrected using the fuzzy filter after two iterations $(N = 2)$.

Figure 10 shows another example of a well log treated with the differential fuzzy filter, after two iterations $(N = 2)$. Compared to the classic median filter with two points, the fuzzy filter significantly reduced noise levels (Fig. 11). In the proposed method, each iteration entails a filtering process, and the fuzzy filter increases or decreases the filtering level, as necessary, to obtain the correct filtering of the considered data.

To more clearly demonstrate the efficacy of the proposed fuzzy filter in comparison to the median filter, Figure 11 shows a highlighted section of the filtered wireline log from Figure 10. After two iterations, the fuzzy filter effectively attenuated the oscillations due to noise by smoothing the peaks over the entire section, while the two-point median filter was unable to remove these oscillations and was only able to lower the peaks in some parts of the well log.

Application to actual data

Figure 12 shows the facies description in a depth window of one borehole drilled in the Namorado field, Campos Basin. Comparing core data with well logs requires some care with regard to sampling and scale; however cores can be used to provide a general overview of the well log data. In Figure 12, the facies description shows a shale package from 3,018 to 3,022 m in depth. In this same depth section, the GR log (Fig. 13) shows values ranging between 70 and 100 UAPI. On a massive shale package, this oscillation can be associated with the presence of noise.

Figure 13 shows the results after two iterations of fuzzy filter and two-point classical median filter. The application of fuzzy filter results in a better quality wireline log because the oscilla-

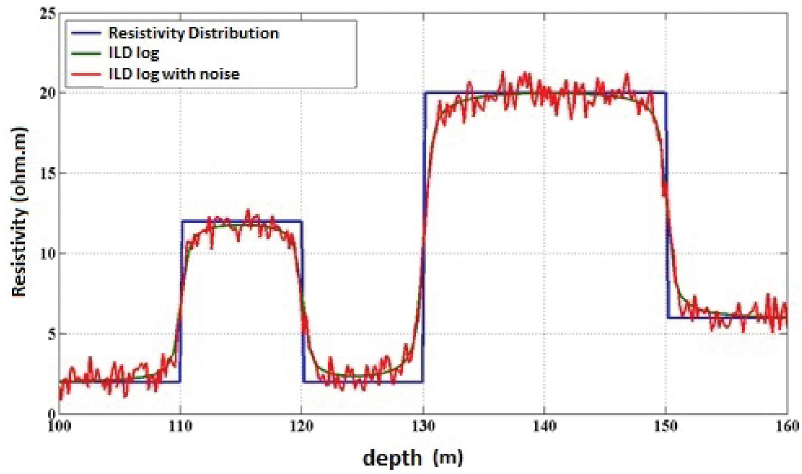

Figure 10 – The synthetic resistivity induction log without noise (green), with noise (red), and showing the distribution of the resistivity (blue).

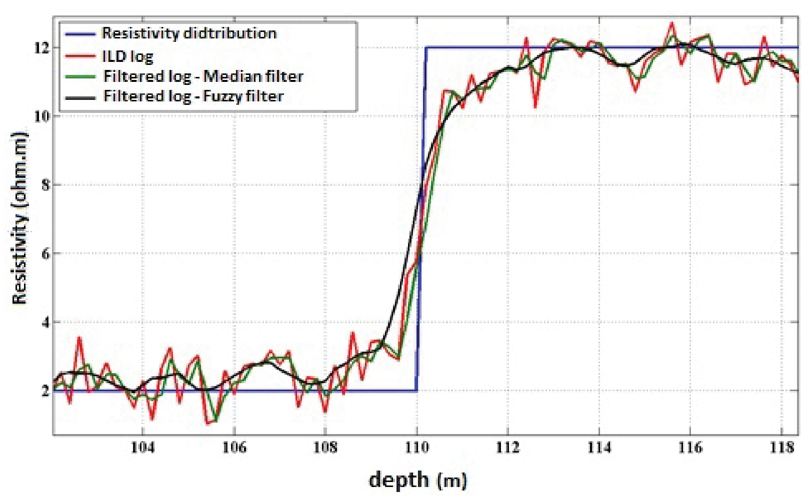

Figure 11 – A section of the conductivity log treated with the two-point median filter and the fuzzy filter after 2 iterations (N=2), showing the distribution of conductivities (blue), resistivity log with noise (red), the log filtered with the median filter (green), and the log filtered with the fuzzy filter (black).

tions observed in the GR log were attenuated. In the subinterval of the 3,018 to 3,022 m depth shows a jump of 10 UAPI with an oscillation of roughly 2 UAPI. The largest jump may represent internal variations of the shale while the smaller oscillation may be due to noise. The fuzzy filter acted on the smaller oscillation, eliminating it and preserving the facies information.

Figure 14 shows the deep induction log treated by the two-iteration fuzzy filter and the two-point median filter. Once again, the application of fuzzy filter results in a better quality wireline log, because it smoothed the signal more efficiently and preserve some discontinuities represented by small peaks at depths of 3,022 to 3,024 m and at 3,036 m.

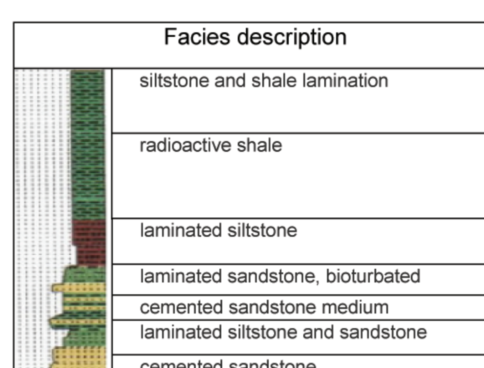

Figure 12 – A section from the facies description of a borehole from the Namorado field, Campos Basin.

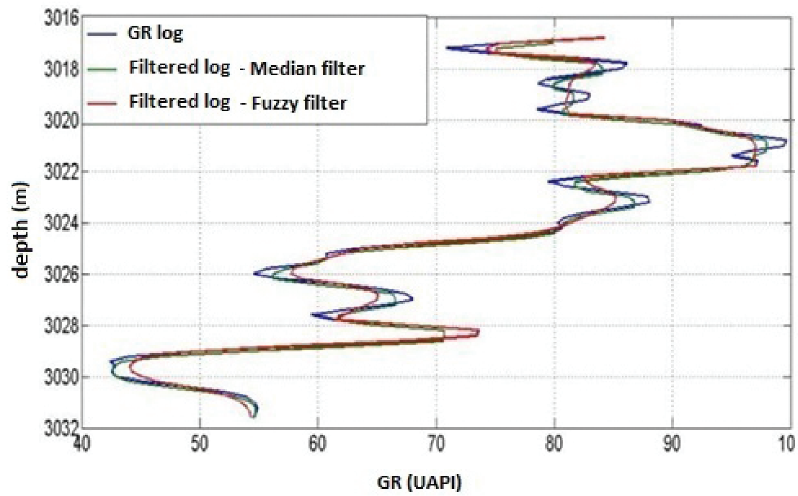

Figure 13 – The gamma ray (GR) log with noise (blue); the well log filtered using the median filter (green); and the well log filtered with the fuzzy filter (red). Facies descriptions are highlighted between depths of 3,018 and 3,022 m.

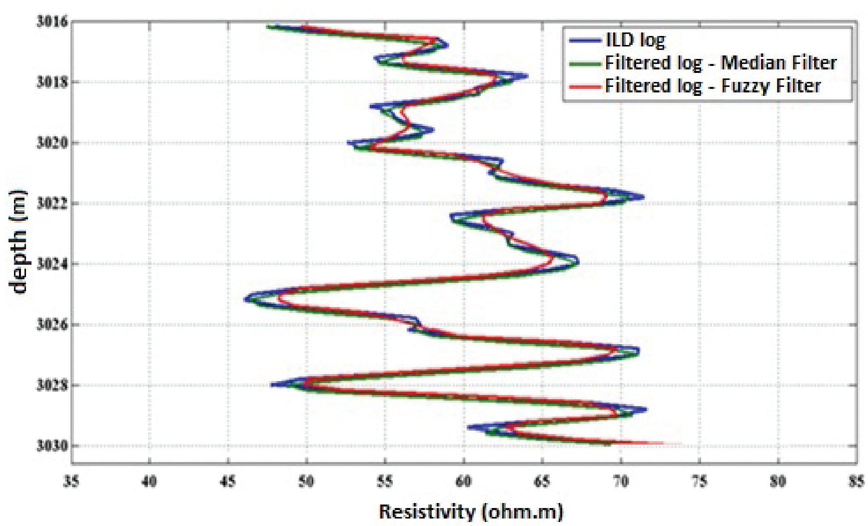

Figure 14 – Induction log (ILD) (blue); the ILD log filtered with the median filter (green); and the ILD log filtered with the fuzzy filter (red).

Applications to borehole images

In Guerra (2004), the simulated sonic log enabled the imaging of the P-wave first breaks using data obtained by receivers or in this case, the semblance distributions. Figure 15a shows a simulation of two elastic half-spaces, **(2)** and **(3)** traversed by a borehole **(f)**. Region **(2)** simulates shale, and region **(3)** simulates sandstone. Through the semblances obtained in the simulation, an image of the normalized amplitude residuals, RAN (Guerra, 2004), can be generated. We note the behavior of the parameter transit time, given by the time when the tool moves through the borehole and crosses the simulated half-space (Fig. 15b). However, if the semblances are noisy, the resulting RAN image may show nothing (see Fig. 15c). Because the noise occurred on the micro-seismograms, processing the RAN image to remove them did produce satisfactory results. When the fuzzy filter was applied to the semblance data, there was significant improvement in the image, which started to reveal a more detailed behavior of the transit times in relation to lithologic changes (Fig. 15d).

Performance evaluation

Synthetic data are ideal for testing the performance of the fuzzy processing algorithm because they allow the control of the entire process (i.e., signal generation, the introduction of noise and the comparison with the initial noiseless data). The model adopted

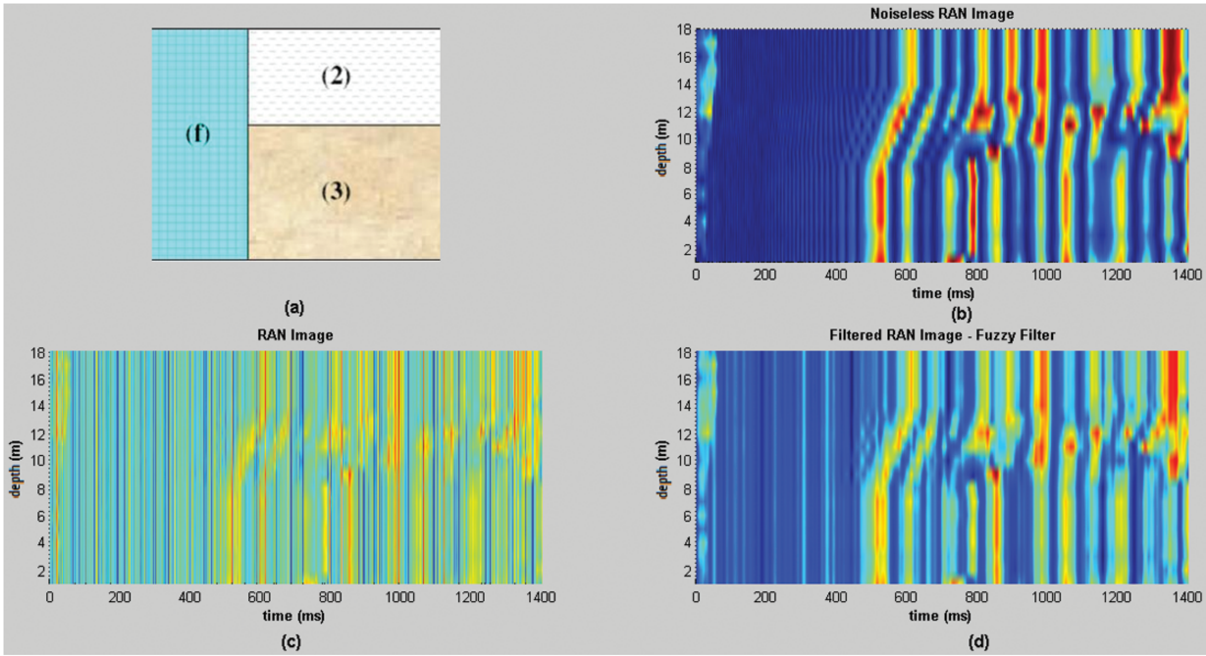

Figure 15 – a) The geological model of well section (f) crossing a shale formation (2) and sandstone (3). b) A RAN image obtained by simulation of the sonic log in panel a. c) The same RAN image with added noise. d) The result of fuzzy filtering of the image in panel c.

for performance evaluation is the same showed in Figure 9. The added noise (Gaussian and random) may be limited in range and in number of points. To evaluate and compare the performance of the fuzzy filter, the MSE (mean square error) was obtained using the following equation:

$$MSE = \frac{1}{N}\sum_{j=1}^{N}(fn_i - f_i)^2. \qquad (20)$$

where fn_i are the values of the original log (noiseless), f_i are the values of the filtered log, and N is the total number of points of the log. A high MSE can indicate a low level of filtering or distance from the original signal. A low MSE can indicate more efficient filtering and good recovery of the original signal. To compare the performance of the fuzzy filter, we applied three types of filtering: median filter, zero-phase filter and the Savitzky-Golay filter (Orfanidis, 2010). Table 1 shows the MSE values used to compare the performance of each filter.

In Table 1, the MSE increases with the percentage of the signal affected by noise, with the exception of the zero-phase and Savitzky-Golay filters, for which the values had few fluctuations. The MSE for the non-linear filters, particularly the median filter, increased with the percentage of the signal affected. The fuzzy filter had more efficient filtering after one iteration than after three iterations, showing that high levels of filtering can damage the signal. Table 2 shows the MSE values for noise levels that are 30% of the maximum amplitude of the ideal signal. Once again, the MSE increases with the percentage of the signal affected by noise. However, the number of iterations plays an important role when noise levels are high. In this case, the three-iteration fuzzy filter had the best performance in comparison to the other filters, especially when a significant percentage of the signal was affected.

Table 1 – MSE values for the noise level, giving the maximum 15% of the greatest absolute value of the ideal log. Values in bold indicate the most efficient filtering levels.

Filter	20%	40%	60%	80%	100%
Med2	0.0582	0.0726	0.0881	0.1053	0.1033
Med4	0.0441	0.0498	0.0632	0.0724	0.0741
Fuzzy1	**0.0209**	**0.0309**	**0.0408**	**0.0577**	**0.0566**
Fuzzy3	**0.0283**	**0.0467**	**0.0645**	**0.0683**	**0.0643**
ZF	0.2578	0.2544	0.2503	0.2575	0.2537
SG	0.6896	0.6822	0.6643	0.6732	0.6714

Table 2 – MSE values for the noise level, giving the maximum 30% of the greatest absolute value of the ideal log. Values in bold indicate the most efficient filtering levels.

Filter	20%	40%	60%	80%	100%
Med2	0.3345	0.5414	0.7288	1.0493	1.0252
Med4	**0.0611**	0.1335	0.2952	0.4602	0.5017
Fuzzy1	0.1230	0.1635	**0.2819**	0.4238	0.4383
Fuzzy3	0.0961	**0.0467**	**0.1873**	**0.2823**	**0.2935**
ZF	0.2620	0.2717	0.3465	0.3952	0.3830
SG	0.6884	0.6936	0.7003	0.7377	0.7357

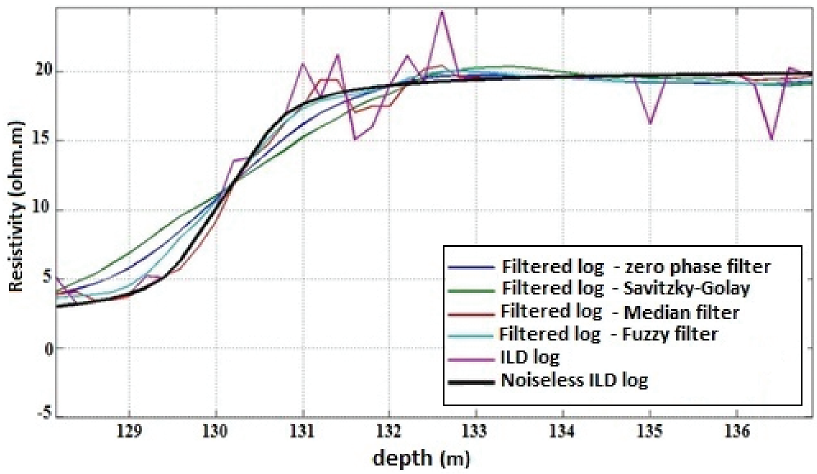

Figure 16 – The deep induction log in detail. The original log is shown in black, and the log after fuzzy filtering is shown in light blue. Compared to the other filters, the fuzzy filter results in a smoother log and good similarity to the original log.

Figure 16 compares the log after different types of filtering. The fuzzy filter performed the best, resulting in the lowest MSE and smoothest log, with smaller perturbations from the original noise-free well log.

CONCLUSION

The fuzzy filter method worked fairly well when applied to synthetic and actual wireline logging data, and it significantly attenuated the noise. Only two iterations were necessary to obtain good results, which shows the convergence of the algorithm. For synthetic data, the number of iterations must be chosen carefully to decrease the noise level and maintain the integrity of the signal, i.e., the lithologic information. This filtering process does not produce shifts in the filtered log. It is important to assess the continuity of the signal and noise, avoid unnecessary filtering, and prevent any loss of signal when defining the differentials as input parameters and inference objects of the fuzzy system. In future work, a better definition of the input parameters, inferences, membership functions, and criteria for choosing the number of iterations will be needed to simplify and optimize the fuzzy filtering algorithm, especially for real data.

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NOTES ABOUT THE AUTHORS

Carlos Eduardo Guerra holds a master's degree in geophysics from the Universidade Federal do Pará – UFPA (1994), and a Ph.D. in geology from the Universidade Federal do Rio de Janeiro – UFRJ (2004). He is currently an adjunct professor at the Universidade Federal do Oeste do Pará – UFOPA, a researcher and external member of the UFPA/Graduate Program in Geophysics, and a coordinator of the Earth Sciences Program at UFOPA/STM and the Remote Sensing Laboratory of Santarém (Laboratório de Sensoriamento Remoto de Santarém – LASERS). He has experience in applied mathematics, modeling, and soft computing, mainly focusing on the following themes: geophysics, well logging, processing and analysis of biomedical imaging, multispectral imaging and remote sensing.

André José Neves Andrade holds a bachelor's degree in civil engineering from the Universidade Federal de Viçosa – UFV (1986), a master's degree in geophysics from the Universidade Federal do Pará – UFPA (1992), and a Ph.D. in geophysics from the Universidade Federal do Pará – UFPA (1996). He is currently an associate professor, level II, at the UFPA. He has experience in the geosciences, with emphasis on wireline logging and he mainly focuses on the following themes: formation evaluation, petrophysics (core analysis), and intelligent algorithms (artificial neural networks, fuzzy inference, and evolutionary methods).

Ivnon José de Faria Lima holds a master's degree in geophysics from the Universidade Federal do Pará – UFPA (2011). He worked in processing and filtering well log data using fuzzy systems.