Brazilian Journal of Geophysics (2022) 40, 1, 83–96
Brazilian Geophysical Society
ISSN 2764-8044
DOI: 10.22564/brjg.v40i1.2134

CORRELATION PROPERTIES IN WORLDWIDE AND SYNTHETIC EARTHQUAKE NETWORKS

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ABSTRACT. In this work, we studied the correlation properties of seismic networks by analyzing the assortativity of worldwide and synthetic earthquake networks. We used data from the World Earthquake Catalog for the period from 2002 to 2016, considering earthquakes with magnitude thresholds 4.5 and 5.0. Shallow earthquakes (a depth of up to 70 km) and deep earthquakes (a depth greater than 70 km) were analyzed separately. Synthetic data were produced from simulations using a modified version of the Olami-Feder-Christensen model, which can reproduce several statistical characteristics of actual earthquakes. The study was carried out for two methodologies of connections between the network elements, where the correlation measures were calculated for all networks. The results for shallow earthquakes and synthetic data indicate: assortative correlation (locations with similar seismic activities tend to have a greater number of connections between them); mainshocks induce other mainshocks in both close and further away regions; the structure found has a type of "attracting dynamics", where the places with a more intense seismic activity produce large numbers of connections in other locations around them. Deep earthquake networks are neutral and therefore do not have an explicit correlation type. Our findings agree with previous works for specific areas and contribute to better understand correlations between seismological regions.

Keywords: earthquakes; complex networks; computational modeling; correlation properties.

INSTRUCTIONS

We are surrounded by several systems that contain a huge number of components in which their elements have many kinds of interactions with each other. These systems belong to the category of "complex systems", making it difficult to understand the dynamics of the entire system knowing only the behavior of the system elements, since the collective behavior is not trivial. In this framework, complex networks are in the heart of complex systems once they have the power to decode the interactions between the system elements, being a powerful tool to investigate the topological structure and statistical behavior of complex systems. The science of complex networks has been successfully applied in many real-world networked systems, such as the Internet, economic market, spread of diseases, solar flares, and social relationships (Watts and Strogatz, 1998; Barabási and Albert, 1999; Albert and Barabási, 2000, 2002; Dorogovtsev and Mendes, 2003; Newman, 2003; Gheibi et al., 2017).

In the last thirty years, several works have implemented the concept of complexity in the study of earthquakes, aiming to better understand and characterize the seismological dynamics and properties (Bak and Paczuski, 1995; Roberts and Turcotte, 1998; Bak et al., 2002; Watkins et al., 2016). The complex network theories started to be used in the seismological study in Abe and Suzuki (2004a, 2004b). The authors constructed earthquake networks for seismic data from California and Japan, taking into account spatial and temporal information of successive earthquakes. After Abe's seminal paper, several authors have been applying similar complex network concepts to earthquake studies (Lotfi and Darooneh, 2012; Ferreira et al., 2014; Pastén et al., 2016; He et al., 2021; Leon et al., 2022). It is noteworthy that over the years, other authors have also studied the earthquake phenomenon under the viewpoint of complex networks, by using approaches different from Abe and Suzuki, such as the Baiesi-Packzuski model (Baiesi and Paczuski, 2004), the visibility graph model (Telesca and Lovallo, 2012), and the space-time influence domain (He et al., 2014). Despite the different network construction methods and analyses, the results found in the various models show that the earthquake networks have similar behavior to the ones of other networks found in nature for many different phenomena.

A handy tool in studying earthquakes is the use of computer simulations. These simulations can be of the type "simplified", where the models capture the main characteristics of the phenomenon, or of the type realistic and more comprehensive, trying to describe the phenomenon in a more detailed way. In statistical studies, it is common to use simplified models capable of generating large amounts of data without requiring substantial computational capacity. The use of complex networks for the statistical study of earthquakes can be done by using actual earthquake data or by using synthetic data generated through computer simulation models. One of the most used simplified models is the one created by Olami, Feder and Christensen (OFC model), which incorporates several characteristics of complex systems (Christensen and Olami, 1992a, 1992b; Olami et al., 1992). A more detailed explanation of this model will be presented in the next section.

Previous works have analyzed seismological data (from actual and synthetic catalogs), performing some of the most common and fundamental features of complex networks, such as the degree distribution, the clustering coefficient, and the average shortest path. However, another interesting characteristic to be studied in earthquakes is the correlation property. Correlation properties can be analyzed in complex networks using a network measure named *assortativity*, which indicates a type of connection preference that elements tend to have when connecting to each other. For example, in social networks, it is observed that people tend to relate to other people belonging to the same group as themselves (Newman, 2003). However, the protein-interaction network of yeast has the opposite property. The proteins with more significant connections interact much more with small-connected proteins (Jeong et al., 2001). In this way, the study of the assortativity correlation feature can help provide more complete information on the structure and dynamics of the system. The correlation property using the assortativity concept will be presented in the theoretical background section of this paper.

Therefore, in order to advance one more step towards greater knowledge about the earthquake phenomenon, we analyze in this paper the assortativity features for the networks created from worldwide seismic events using two different models of connections for two different datasets: one for shallow earthquakes and the other for deep earthquakes. We differentiate the seismic events concerning their depths since shallow and deep earthquakes are mechanically different from each other (Frohlich, 1989, 2006). Still, deep earthquakes occur, in general, in subduction zones of tectonic plates (Frohlich, 2006). On the other hand, shallow earthquakes are not only related to these zones, but they also happen in faults and plate slippage, making their occurrence more distributed in geographic regions around the world than deep seismic events.

Therefore, we make the data division by depth to observe whether it would exist differences in the properties of the networks created with these two datasets. We adopted the division used in Frohlich (1989) and Spence et al. (1989): shallow earthquakes are those with a depth of up to 70 km, and deep earthquakes are the ones located deeper than this value. Furthermore, a network of connections was also created using earthquake data simulated with a modified version of the Olami-Feder-Christensen model (Ferreira et al., 2015). The results were compared with those of the actual seismic events.

This paper is organized as follows. First, we have a brief theoretical background on complex networks and the OFC model. The following section is dedicated to presenting information about our worldwide and synthetic earthquake data catalogs. Then, the methods employed to construct our networks are described. Finally, we show and discuss the results obtained and present the conclusions.

THEORETICAL BACKGROUND

Aiming to make this paper self-contained, we present a brief review of theoretical topics considered important to the development of this article.

Fundamental Concepts of Complex Networks

Networks are commonly represented by graphs, where the elements are nodes (or vertices), and the interactions are expressed by links (or edges) between the nodes. A fundamental property in networks is the degree, k_i, of each node, i, which gives the number of links between the node i and other nodes. The links in a network can be directed (if the links have a specific direction, going from one node to another) or undirected (if there is no direction for the links between the nodes), as shown in Figure 1. In a directed network, there is a distinction between the number of incoming links, expressed by the incoming degree, k_i^{in}, and the outgoing links, given by the outgoing degree, k_i^{out}. In this way, the total degree in directed networks is provided by $k_i = k_i^{in} + k_i^{out}$. Examples of directed networks are the World Wide Web (WWW), where the links point from one webpage to another, and an ecological web, where the directed links indicate which animal is a predator of the

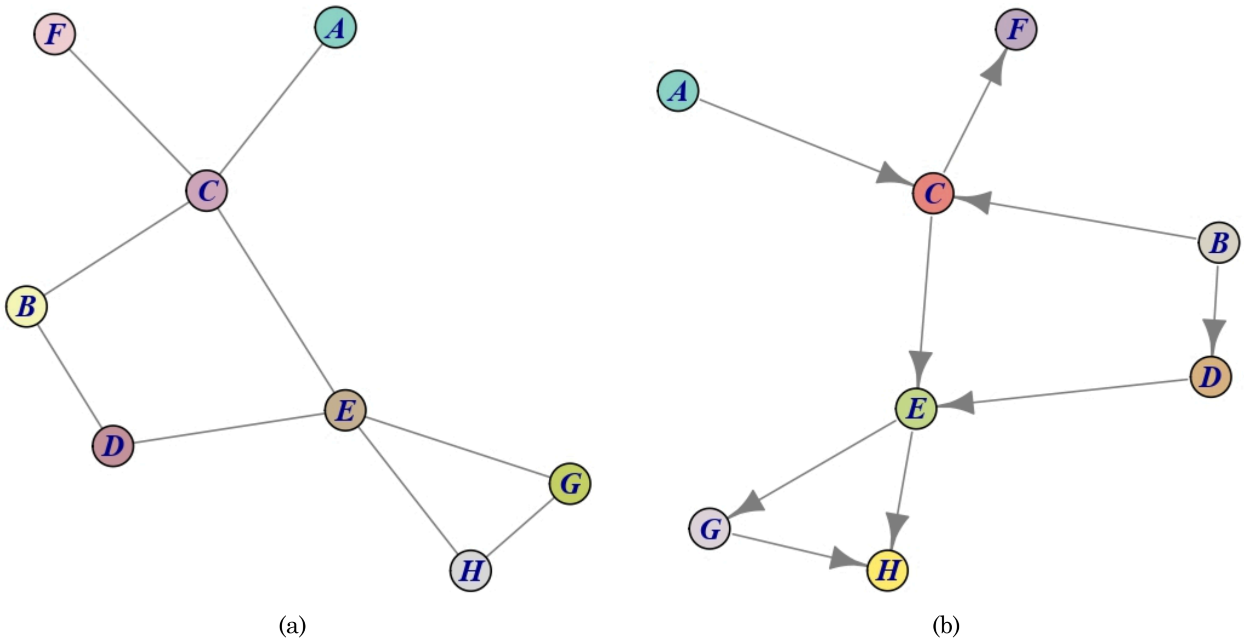


Figure 1: Examples of (a) an undirected network and (b) a directed network. The networks have 8 nodes and 9 links each one.

other. Undirected networks can be exemplified by a scientific collaboration, since if a scientist "A" collaborates with a scientist "B", the scientist "B" also collaborates with "A".

For a mathematical description, the networks are usually described by *adjacency matrices*. If an unweighted network (i.e., all links have the same weight) has N nodes, the links between them are represented by a $N \times N$ matrix, with elements:

$$A_{ij} = \begin{cases} 1, & \text{if there is a link from } i \text{ to } j \\ 0, & \text{if there is no link from } i \text{ to } j. \end{cases}$$

In an undirected network case, the adjacency matrix is symmetric, which means, $A_{ij} = A_{ji}$.

From the nodes' degrees, it is possible to calculate the network's *degree distribution* $P(k)$, which is the probability that a randomly selected node in the network has a degree k. Since the functional form of $P(k)$ is deeply related to many properties in the network, it assumes an essential role in network studies. One remarkable example is that if the degree distribution of nodes follows a power law, i.e., $P(k) \sim k^{-\gamma}$ (where γ is a positive constant), it represents a signature of an organizing principle, called *scale-free* property. The presence of a power-law for the degree distribution, instead of a Poisson distribution, implies that the nodes are not connected randomly; instead, they follow a preferential attachment rule to connect to each other. It means that a scale-free network has several small degree nodes and only a few with high degree values. These last ones are named *hubs*.

Another important feature of a complex network arises when every node is "close" to every other node, i.e., it is possible to go from one arbitrary node to another taking only a few "steps". When it happens, these networks are called *small-world* networks. As proposed by Watts and Strogatz (1998), two important metrics to classify a small-world network are the clustering coefficient and the average path length. The clustering coefficient measures how likely it is that two elements connected to a node i are also connected to each other. The average path length is the average of the shortest distances between all pairs of nodes in the network, where the distance between two nodes is the number of links between them, as exemplified in Figure 2. Small-world networks have a high clustering coefficient value compared to a random network with the same characteristics and a small average path length compared to the number of nodes in the network.

Within this frame, the complex networks approach has been applied to seismological studies from many areas in the world, as Japan, California, Iran, Chile, Greece, and worldwide (Abe and Suzuki, 2004a, 2004b; Lofti and Darooneh, 2012; Ferreira et al., 2018; Pastén et al., 2018; Chorozoglou et al., 2019). The results found in the previous works show the scale-free and small-world features in the networks built from the seismological data, where the degree exponents γ for the networks are within the usual range $2 \leq \gamma \leq 3$, which is the range for several of the existing networks in nature (Albert et al., 1999; Barabási, 2002; Ebel et al., 2002). As indicated by Abe and Suzuki (2006a), the presence of scale-free properties in earthquake networks indicates that, geographically, aftershocks associated with a mainshock tend to return to the locus of the mainshock, creating hubs similarly to the preferential attachment rule. Furthermore, the small-world feature indicates that the earthquake network is significantly clustered, and the average distance between two arbitrary nodes (geographical regions where earthquakes have occurred) is very small.

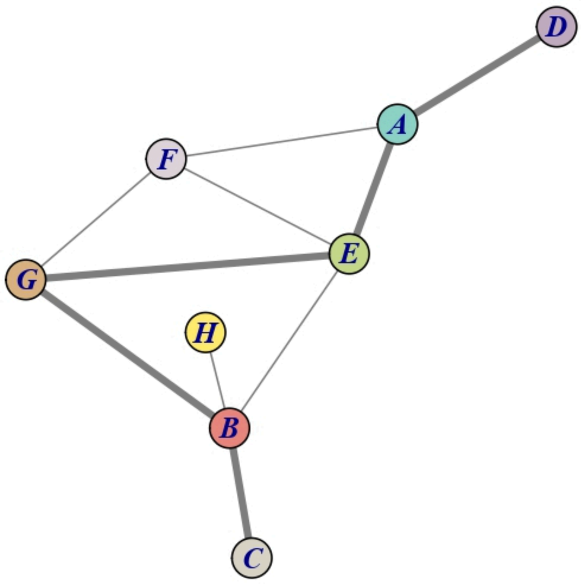


Figure 2: The highlighted links exemplify one possible path between nodes *C* and *D*. However, the shortest path between these nodes is the one that follows the nodes sequence *C-B-E-A-D*.

Correlation Properties in Complex Networks

In the last twenty years, the study of correlation properties in networks using assortativity has been implemented in many real-world networks (Newman, 2002, 2003; Catanzaro et al., 2004; Foster et al., 2010; Johnson et al., 2010; Piraveenan et al., 2010; Gheibi et al., 2017). Assortativity (or assortative mixing) refers to the tendency of nodes in a network to connect to other nodes with similar properties, making the links not to be placed between nodes completely at random, but depending in some way on the property in question. Here, we focus on assortativity in terms of a node's degree. The analysis of this property allows us to investigate the relation between the connectivity degrees of the nodes that link to each other. A statistical measure that is commonly used to analyze this preference is the nearest-neighbors average connectivity of nodes, or also denominated degree correlation function (Pastor-Satorras et al., 2001; Vázquez et al., 2002; Barabási and Pósfai, 2016), expressed as

$$k_{nn}(k) = \sum_j jP(j|k) \tag{1}$$

where $P(j|k)$ is the conditional probability that an arbitrary selected edge links a j-degree node with a k-degree node. This function considers the average degree of the neighbors of a node as a function of its degree k. If it is independent of k, the network has no obvious degree correlation and is called neutral. When, however, $k_{nn}(k)$ increases with k, the network is assortative. It means that the hubs (nodes with high degrees) of the network tend to connect to other hubs and nodes with low degrees tend to be linked to other low degree nodes. On the other hand, if $k_{nn}(k)$ decreases with k, the network is disassortative, i.e., the hubs prefer to link to nodes with low degrees (Barabási and Pósfai, 2016).

Thus, the degree correlation function can help detect the presence or absence of correlations in real networks. However, a helpful way to capture the magnitude of the correlations present in the networks is using a unique number. This number can be computed from the fitting of the $k_{nn}(k)$ plot or by calculating the degree correlation coefficient, defined as follows.

The degree correlation coefficient, which is the Pearson correlation coefficient between the degrees found at the two ends of the same link, is a complementation of the analysis of the degree correlation function and provides us with a quantitative characterization. We calculate this coefficient by

$$r = \sum_{jk} \frac{jk(e_{jk} - q_j q_k)}{\sigma^2}, \tag{2}$$

where e_{jk} is the probability of finding a node with degrees j and k at the two ends of a randomly selected link; q_k is the probability of existing a node with degree k at the end of a randomly selected link and

$$\sigma^2 = \sum_k k^2 q_k - \left[\sum_k k q_k\right]^2, \tag{3}$$

is the variance of q_k. The value of r varies from -1 (perfect disassortativity) to 1 (perfect assortativity). If $r = 0$, the network has no assortative (or disassortative) mixing and, therefore, is neutral.

Previous works have considered the assortative mixing approach to analyze the correlation properties in earthquake networks for some regions, such as California, Japan and Iran (Abe and Suzuki, 2006a; Lofti and Darooneh, 2012; He et al., 2021). These studies found an assortative behavior for the earthquake networks.

The OFC Model

The model created in 1992 by Olami, Feder and Christensen (OFC model) can reproduce several statistical properties of earthquakes (Olami et al., 1992). This model is widely used because, despite its apparent simplicity, it can reproduce several characteristics found in actual seismological data, such as the Gutenberg-Richter law.

Standard OFC model

The standard OFC model can be represented by a bidimensional square $\ell \times \ell$ lattice with $N = \ell^2$ blocks (sites) interconnected to its first neighbors by springs. Each block is connected through a spring to a single rigid driven plate and by friction to another rigid fixed plate on which they stay. This blocks arrangement represents a regular topology of the lattice. Due to the relative motion between the plates (imposed by the model), all the blocks will be subjected to an elastic force which tends to put them in motion, and to other frictional forces, opposite to the first. When the resulting force in one of the blocks is greater than the maximum static friction force, the block slides and relaxes to a position of zero force, and a fraction (α) of its tension is equally redistributed between its nearest neighbors. It produces a rearrangement of forces in its first neighbors, which can cause other slippages and the emergence of a chain reaction. The first block to move is the earthquake's epicenter, and the magnitude s of this earthquake is measured by the number of blocks that skidded.

When an earthquake finishes, the continuous relative movement between the plates will cause a force accumulation in all blocks. In this way, the slip of a block will occur after some time, and a new earthquake process begins. The model assumes that the time interval between two earthquakes is considerably longer than the duration of an earthquake itself.

Small-world-like OFC model

As verified in Ferreira et al. (2015), a better agreement with real data is obtained when the lattice's topology in the OFC model is not regular, but, instead of that, follows the rule of topology construction proposed by Caruso et al. (2006), based on the Watts–Strogatz mechanism to generate small-world networks (Watts and Strogatz, 1998). Starting from the regular two-dimensional topology, the rule of construction consists of taking each edge of the lattice (the spring that connects the blocks) and randomly reconnecting it to other blocks with probability p (rewiring probability), keeping fixed each block's original number of connections. That mechanism generates a *small-world-like* lattice for the OFC model, in which the resultant lattice has a topology between the regular and the random ones.

The use of a small-world-like topology for the OFC topology makes the system more realistic since it allows more effective long-range interactions between the blocks, agreeing with several previous works which indicate spatial and temporal long-range interactions between earthquakes (Kagan and Jackson, 1991; Stein, 1999; Mega et al., 2003; Tosi et al., 2008; Toda and Stein, 2020).

Thus, in our studies, we used the OFC model with a small-world-like topology to create synthetical data catalogs to calculate spatial and temporal distributions and compare them to those produced by real data catalogs of earthquakes.

DATA

In this study, we used natural and synthetic earthquake data catalogs.

Worldwide Earthquakes

The dataset was obtained from the World Catalog of Earthquakes of the Advanced National Seismic System (ANSS)[1], and it covers earthquakes from the entire world between 2002 and 2016. We considered the magnitude types M_b (body-wave magnitude), M_L (local magnitude) and M_w (moment magnitude). For the record, we only considered earthquakes with magnitude (m) larger or equal to 4.5 because, in that catalog, the events with magnitudes less than 4.5 are not completely registered for the whole world. The total of events is 101746, of which 80520 are shallow earthquakes (earthquakes with a depth of up to 70 km), and 21226 are deep earthquakes (events occurred at depths greater than 70 km).

These data had a good agreement with the Gutenberg-Richter law (GR) (Gutenberg and Richter, 1942), with a b-value exponent equal to 1.080 ± 0.003 for the shallow seismic events, and 1.080 ± 0.010 for the deep ones. In concordance with many previous works (Abercrombie, 1996; Kanamori and Brodsky, 2004; Ahmed et al., 2016; Fiedler, 2018), given that in our data we consider only earthquakes with magnitude greater than or equal to 4.5, the b-value close to 1.0 is a consistent result for earthquakes worldwide.

We have not distinguished mainshocks from aftershocks in our data in this work. Helmstetter et al. (2005) and Marsan and Lengliné (2008) discuss in their studies the relevance of small earthquakes and aftershocks in triggering other earthquakes, where it is observed that these seismic events appear to be of major importance in the earthquakes interactions. Therefore, as we aim to study how earthquakes worldwide are correlated, it makes sense that we do not eliminate aftershocks from our datasets. Moreover, this approach was also adopted in Bak et al. (2002), Davidsen and Paczuski (2005), and Abe and Suzuki (2006b).

Synthetic Earthquakes

For the process of earthquake nucleation (i.e., how the earthquakes are generated) we have used the small-world-like OFC model, which generates a synthetic seismological catalog. In our simulations, we have used *non-periodic open boundary* conditions. This condition imposes that all blocks in the OFC lattice have the same parameter α, independently of their location on the lattice. With that condition, the blocks located at the lattice boundary will have a more dissipative behavior than those in bulk since the elements in bulk are connected to 4 neighbors, and the elements in the borders are connected to only 3 or 2.

Previous studies showed that for the OFC model to have a lattice with small-world-like features and critical behavior, the rewiring probability p must be in a range of values $[10^{-3}, 10^{-2}]$ (Caruso et al., 2006; Ferreira et al., 2015). Thus, our analysis was conducted using $p = 0.001$ for a lattice of size $\ell = 400$, a dissipation coefficient $\alpha = 0.20$, and the number of events generated was 2×10^7 after the transient regime. It is relevant to highlight that we have excluded the earthquakes with magnitude $s = 1$ from the construction of the network because these events seem to obey their own statistics (Grassberger, 1994).

[1] https://earthquake.usgs.gov/data/comcat/

METHOD

The earthquake prediction is one of the goals of a seismological research. However, before that, it is necessary to better understand the earthquake dynamics and possible correlations between different earthquake events and locations. Accordingly, it makes sense to look for interactions due to long-range correlations since, as mentioned earlier, several authors have pointed out in previous works the existence of this kind of interaction, meaning that this characteristic cannot be neglected when studying seismological data.

Therefore, intending to study and search for possible correlations in global earthquakes, Ferreira et al. (2014) constructed a network of worldwide epicenters. Similar to the studies for specific regions, they found that the global network is also scale-free and small-world, showing evidence of long-range correlations across the planet.

To construct the network of global epicenters, following the definition used in Ferreira et al. (2014), we divided the planet in this work into equal square cells (sites) of size $L \times L$, with $L = 20\ km$. A cell becomes a node of the network every time the epicenter of an earthquake is located therein. Besides that, we used the two methods described below to create the links between the nodes.

Successive Model

This methodology was created by Abe and Suzuki (2004a, 2004b) and employed in Ferreira et al. (2014). It consists of connecting a node to its successive one in the temporal order by a directed link. Thus, in this model, the construction of the network considers that each earthquake is related to the one that happens right after it in the temporal series, regardless of the time difference between them.

Time Window Model

In Ferreira et al. (2018), a refined model, called the "time window" model, was created to construct networks of epicenters from all around the globe, which improves the previous successive methodology, and showed evidence of being a better approach to building networks of earthquakes than the successive one. It consists of defining a *time window*, with size T, placed on the chronologically ordered data to create the links between the nodes, where the nodes correspond to the cells where the epicenter of an earthquake has occurred. In this way, the first node inside the window is connected to all other nodes within that window by directed links. After that, the window is moved forward to the next event, and the connections procedure is repeated. Figure 3 illustrates an example of this process. As it can be seen, the time window T works as a temporal filter to connect the nodes.

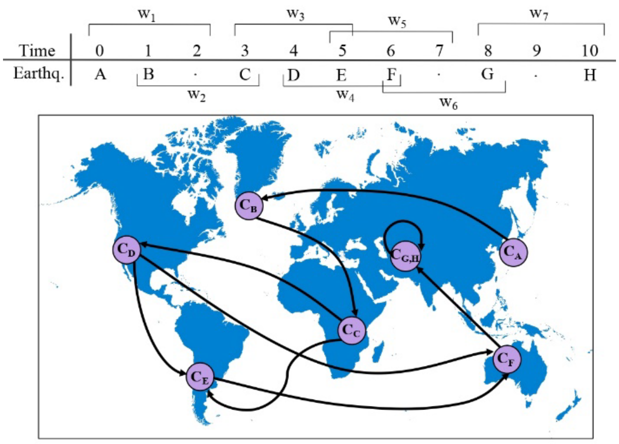


Figure 3: Example of the network's construction for the time window model. The time windows are represented by w_i, where i is the window number, and all the time windows must have the same value (in this example, $T = 2$, in arbitrary units). Events in the same window are connected, as explained in the text. We can see that there are 8 earthquakes (*A, B, C, D, E, F, G, H*), but the epicenters network has only 7 nodes (C_A, C_B, C_C, C_D, C_E, C_F, C_G), because $C_G = C_H$. It can also be observed that the link between C_G and C_H is a self-link.

We built networks using both the successive and the time window models for the shallow and deep earthquakes collected. The time window values were T = 3800 s for shallow seismic events and T = 16500 s for deep earthquakes. These values are the same calculated and used in Ferreira et al. (2018, 2020), respectively.

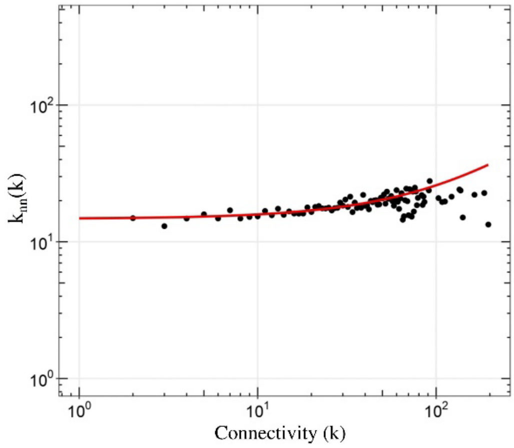
Regarding the data generated with the OFC model, each epicenter was defined as a node. We constructed a network using the successive model of connections, as it was done in Ferreira et al. (2015).

RESULTS

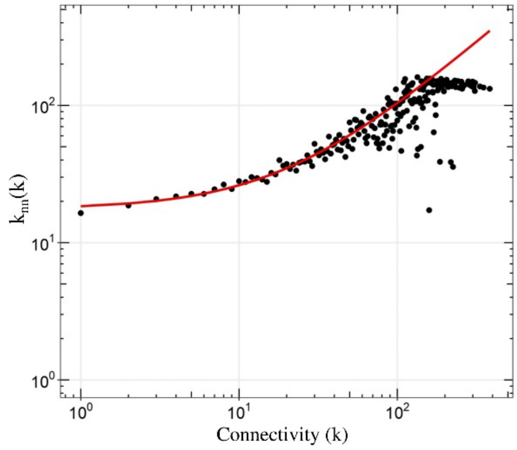
Degree Correlation Function

From equation (1), we calculated the degree correlation function of our networks, using the degree k of the nodes. Figure 4 shows a comparison between the network of shallow earthquakes (m_{th} = 4.5) built using the successive model and the time window model. It is observed that in both distributions the nearest-neighbors average connectivity of nodes, $k_{nn}(k)$, increases linearly with k, which means that these networks are assortative. Therefore, the nodes with a high degree connect on average to nodes with a high degree. This result was the same found in networks of earthquakes from California and Japan (Abe and Suzuki, 2006a), which makes sense since most earthquakes that occur in these areas have depths of up to 70 km (shallow earthquakes).

However, the network constructed with the time window model is more assortative than the one built with the successive model. It is interesting because



(a)



(b)

Figure 4: Nearest-neighbors average connectivity of nodes $k_{nn}(k)$ for the network of shallow earthquakes, for $m_{th} = 4.5$, using (a) the successive model and (b) the time window model. It can be observed that both distributions follow a crescent linear fit (red line). These plots show that both networks have assortative mixing, being the network constructed with the time window model much more assortative.

Ferreira et al. (2018, 2020) showed that the time window model gives results that make more sense than the successive model (e. g., it naturally identifies the world's places with more occurrence of seismic events). The high assortative value found implies that areas of the world with intense shallow seismic activity are not only correlated but strongly correlated.

It is also interesting to note that in Figure 4 the linear growth does not hold for high degree-nodes. This behavior is expected in scale-free networks, like ours, since the system is unable to sustain assortativity for high-degree because there are only a few nodes with large values of degree (hubs). The same happens for other real networks, as citation networks.

Moreover, in the global catalog from which we collected our data, there are several events with a "fixed depth" equal to 10 km,[2] when there is no certainty about the actual focus depth. Thus, to analyze if this value of depth is biasing our results for the shallow earthquakes, we removed them and plotted the nearest-neighbors average connectivity of nodes $k_{nn}(k)$ for the successive and time window models for the remaining data. As we can see in Figure 5, both distributions have a behavior very similar to those considering all depths (Figure 4), which means that the assortative behavior of the networks of epicenters for shallow earthquakes is not dependent of this value of depth, being, in fact, a property of shallow earthquakes.

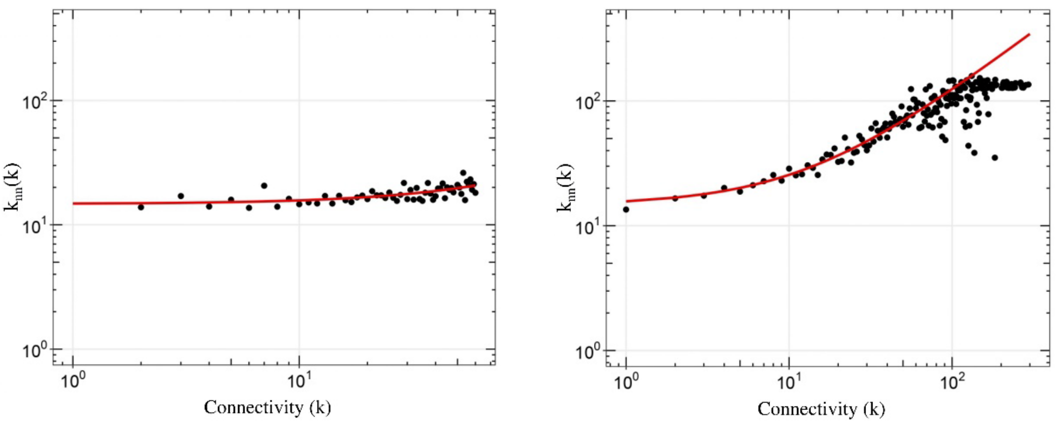
We have also analyzed the shallow seismological data in respect to its magnitude threshold m_{th} to check if the consideration $m_{th} = 4.5$ is satisfactory and if this value influences our results. Figures 6(a) and 6(b) show the nearest-neighbors average connectivity of nodes, $k_{nn}(k)$, for the successive and time window models, respectively, using $m_{th} = 5.0$. The distributions show a linear growth between $k_{nn}(k)$ and k, similarly to what we have found when considering $m_{th} = 4.5$. Therefore, these networks are assortative, independently of the minimum magnitude value adopted.

The results for the networks of deep earthquakes with $m_{th} = 4.5$ are shown in Figure 7. In both cases, successive and time window models, the networks are neutral, i.e., $k_{nn}(k)$ is independent of k. It means that the world's geographical regions with greater deep seismic activity are correlated both with each other and with areas of less occurrence of deep earthquakes, without preference.

Following the same analysis for the shallow seismological data, we analyzed the epicenter networks' assortativity of deep earthquakes using $m_{th} = 5.0$. For the network created using the successive model of connections, no correlation was observed between $k_{nn}(k)$ and k; thus, this network is neutral (Figure 8[a]), as it was found when considering $m_{th} = 4.5$ (Figure 7[a]). On the other hand, in the case of the network constructed with the time window model, $k_{nn}(k)$ increases linearly with k, indicating that this network has assortative mixing (Figure 8[b]), which differs from the result obtained for $m_{th} = 4.5$ (Figure 7[b]). However, when comparing the distributions presented in Figures 7 and 8, we note a significant reduction in the number of nodes for the network built using $m_{th} = 5.0$ (this decrease in the number of nodes can also be seen in Table 1). Then, in this case, the results found for deep earthquakes with $m_{th} = 5.0$ cannot be considered very consistent since the low number of nodes does not allow us to perform good statistics.

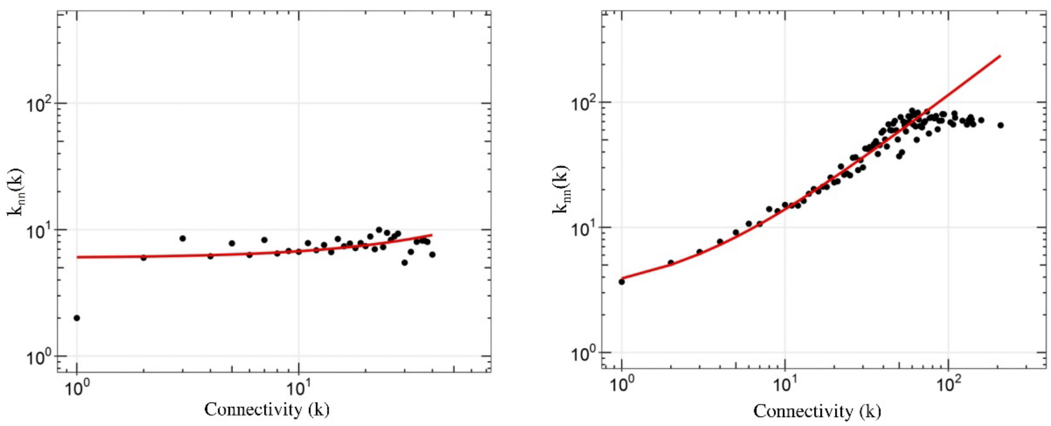
Finally, as shown in Figure 9, we found that the nearest-neighbors average connectivity of nodes $k_{nn}(k)$ for the network of earthquakes simulated with the modified OFC model has an increasing behavior with k, in agreement with the results found for shallow earthquakes. Similar results were also found in a previous work conducted for networks built using the standard OFC model (Peixoto and Prado, 2006).

[2]https://www.usgs.gov/faqs/why-do-so-many-earthquakes-occur-depth-10km



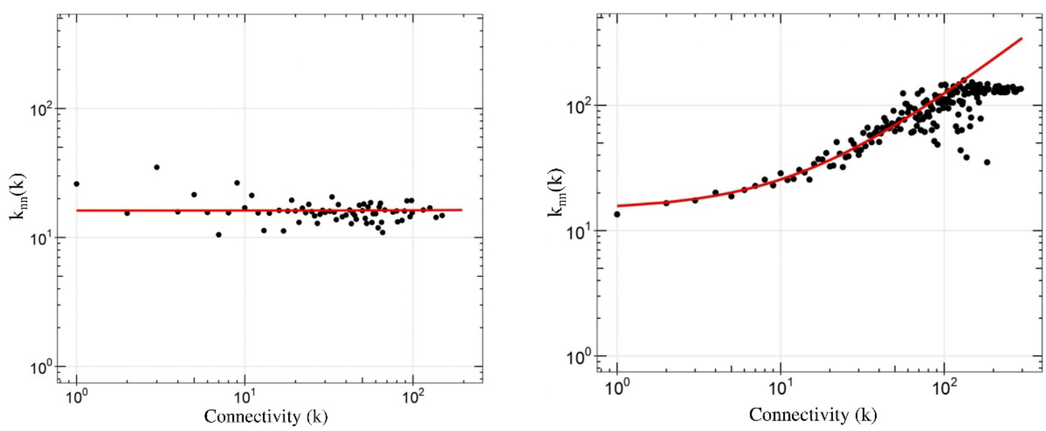
(a) (b)

Figure 5: Nearest-neighbors average connectivity of nodes $k_{nn}(k)$ for the network of shallow earthquakes, for $m_{th} = 4.5$ and without depth equal to 10 km. The networks were constructed with (a) the successive model and (b) the time window model. Both distributions follow an increasing linear fit (red line), which means that these networks are assortative. Again, the network constructed with the time window model is much more assortative.



(a) (b)

Figure 6: Nearest-neighbors average connectivity of nodes $k_{nn}(k)$ for the network of shallow earthquakes, for $m_{th} = 5.0$, created with (a) the successive model and (b) the time window model. By the crescent linear relation between $k_{nn}(k)$ and k (red line), these networks have assortative mixing, being the time window network much more assortative.



(a) (b)

Figure 7: Nearest-neighbors average connectivity of nodes $k_{nn}(k)$ for the network of deep earthquakes, $m_{th} = 4.5$, constructed with (a) the successive model and (b) the time window model. No correlation between $k_{nn}(k)$and k is presented in the distributions, which means that both networks are neutral.

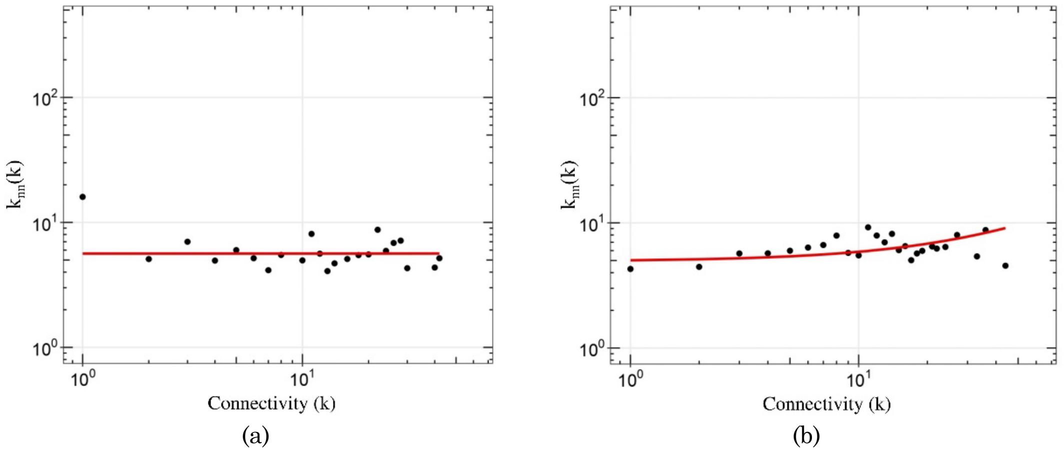


Figure 8: Nearest-neighbors average connectivity of nodes $k_{nn}(k)$ for the network of deep earthquakes, m_{th} = 5.0. In (a) the network was constructed with the successive model. No correlation between $k_{nn}(k)$ and k is presented, showing that this network is neutral. (b) presents the $k_{nn}(k)$ distribution of the network created using the time window model. The distribution shows an increasing behavior, indicating that this network is assortative. The explanation of this difference is in the body of the text.

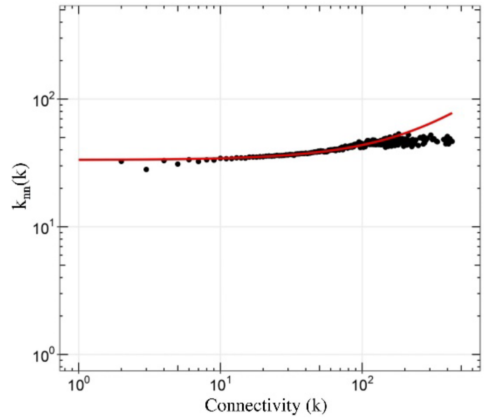


Figure 9: Nearest-neighbors average connectivity of nodes $k_{nn}(k)$ for the network of earthquakes generated with the modified OFC model using the successive model. This network has $k_{nn}(k)$ linearly increasing with k (red line); therefore, it is assortative.

Degree Correlation Coefficient

In addition, to obtain a quantitative result from equation (2), we have calculated the degree correlation coefficient (r) for each of our networks, and the values are shown in Table 1. As seen by the positive values obtained, the networks of shallow events (for both successive and time window models) are assortative in all cases, i.e., considering data for m_{th} = 4.5 with all depths; for m_{th} = 4.5 excluding the fixed depth of 10 km; and for m_{th} = 5.0 with all depths. For these three cases, considering each model separately, the values of r are very close. Moreover, once again, we have noticed that the network of shallow earthquakes constructed with the time window model is much more assortative. It is noteworthy that the r values found for this network are consistent with a similar finding for Japan data, using a different technique from ours (Tenenbaum et al., 2012). As the Japan region has a predominance of shallow earthquakes, our results make sense.

The networks of deep earthquakes with m_{th} = 4.5 for both models, successive and time window, present $r \approx 0$, indicating that they are neutral, as found in the nearest-neighbors average connectivity of nodes ($k_{nn}(k)$) analyses. Additionally, as we observed in the $k_{nn}(k)$ study, a difference was found when looking for the degree correlation coefficient r for deep earthquakes with m_{th} = 5.0. While the network constructed with the successive model presents $r \approx 0$, the one created with the time window model has $r > 0$. However, as shown in Figure 8 and Table 1, when we consider only magnitudes greater than or equal to 5.0, the number of nodes in the networks of deep earthquakes is much smaller than for m_{th} = 4.5 (Figure 7 and Table 1). Consequently, the results indicated by the value of r for deep earthquakes with m_{th} = 5.0 are inconsistent due to the low number of nodes.

Table 1 - The number of nodes (N) and the degree correlation coefficient (r) values of the worldwide and synthetic earthquake networks used in this study.

Network	**N**	r
Shallow earthquakes, m_{th} = 4.5 (successive)	28471	0.0711
Shallow earthquakes, m_{th} = 4.5 and depth equal to 10 km excluded (successive)	20222	0.0809
Shallow earthquakes, m_{th} = 5.0 (successive)	12323	0.0774
Shallow earthquakes, m_{th} = 4.5 (time window)	23380	0.508
Shallow earthquakes, m_{th} = 4.5 and depth equal to 10 km excluded (time window)	14737	0.510
Shallow earthquakes, m_{th} = 5.0 (time window)	6184	0.582
Deep earthquakes, m_{th} = 4.5 (successive)	8958	0.000763
Deep earthquakes, m_{th} = 5.0 (successive)	3086	0.0163
Deep earthquakes, m_{th} = 4.5 (time window)	7675	0.0152
Deep earthquakes, m_{th} = 5.0 (time window)	2478	0.0944
OFC model earthquakes	115510	0.0750

The positive degree correlation coefficient found for the network of earthquakes built from the modified OFC model shows assortativity, as seen in Table 1. This result is in concordance with our findings for the nearest-neighbors average connectivity of nodes $k_{nn}(k)$.

DISCUSSION

To better understand the triggering of earthquakes and their dynamics, it is of great relevance to study how they correlate to each other worldwide. As the authors show in Marsan and Lengliné (2008), knowing how the temporal and spatial domain link together is important to understand the diffusion processes in the earthquake system.

The assortative correlation exhibited in shallow and synthetic earthquake networks is an exciting result. As the hubs are connected to other hubs in earthquake networks, the regions of the world where large earthquakes occurred (e. g. Japan, Sumatra, and Chile) tend to, on average, be linked to each other. It means that the regions where mainshocks occurred are connected to other regions where other mainshocks also appeared, suggesting that mainshocks may induce other mainshocks. Figure 10(a) presents the geospatial image of the network of shallow earthquakes ($m_{th} = 4.5$) created with the time window model. Despite Figure 10(a) displaying only the 2% of nodes with the highest degree of the network, they hold 16% of the links of the entire network. It is also possible to observe that hubs are not connected only to other close hubs (as in the case of Japan, which holds more than one hub), but they are also linked across the planet.

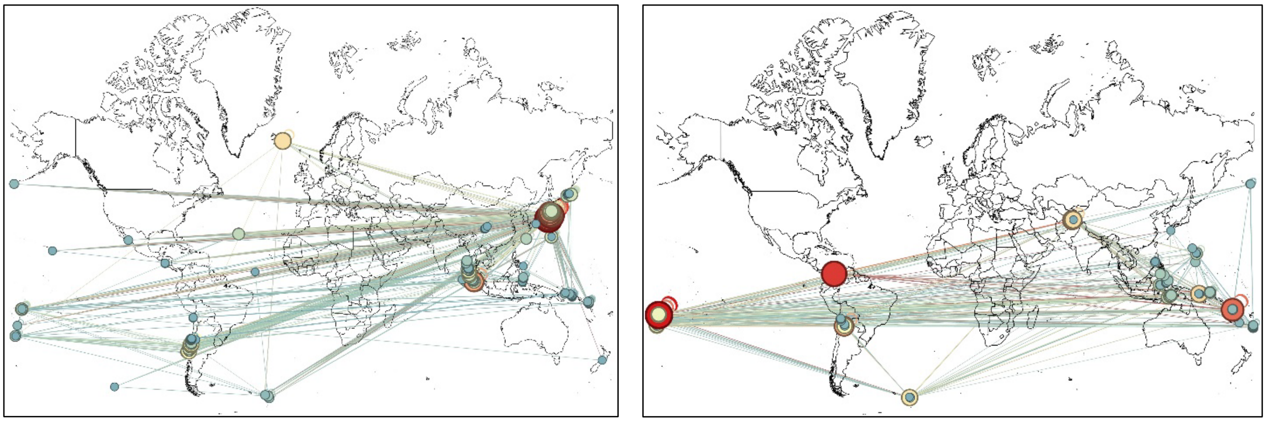
Another point is that removing high-degree nodes in assortative networks is a relatively inefficient strategy for destroying the network connectivity, since removing a hub does not cause significant damage to the network because the hubs form a core group (Barabási and Pósfai, 2016). Consequently, being the earthquake network assortative, even if a seismically active region does not have earthquakes for some time, this will not influence the global seismic dynamics since the network structure will not undergo significant changes.

Besides that, the assortative earthquake networks also reveal a kind of "attracting dynamics", where many hubs tend to be located near each other, i.e., large events tend to produce many hubs in a relatively small region. However, as depicted in Figure 10(a), these regions are not isolated from the rest of the network; instead, they are also connected to hubs in other regions.

Furthermore, analyzing the networks themselves, the assortative property makes the giant component of the networks unable to reach large values. Once the giant component is the largest subset of nodes in the network (where each of its nodes must be connected to at least one other node), the assortative result means that, in earthquake networks, the regions with higher degrees are forced to connect much more to each other, than to the ones with small degree, making the hubs have more probability of belonging to the giant component than to the disconnected parts of the network.

A further interesting point is an agreement between the results for the small-world-like OFC model and for shallow earthquakes, which strengthens the credibility of the OFC model for the study and statistical description of earthquakes. Given the importance of using theoretical-computational models in earthquake research, the validation of a model such as the OFC, which does not need large computational requirements, is an interesting result once it allows inferring characteristics of actual earthquakes from those found using the model.

In the case of deep earthquakes, the neutral behavior found in the networks indicates that the earthquakes correlate at random since, in neutral networks, the nodes are linked arbitrarily (Barabási and Pósfai, 2016). The geospatial image of the network of deep earthquakes, with magnitude threshold $m_{th} = 4.5$, constructed with the time



(a) (b)

Figure 10: Geospatial images of the networks constructed with the time window model between 2002 and 2016, for (a) shallow earthquakes and (b) deep earthquakes, considering $m_{th} = 4.5$. The 2% of nodes with the highest degrees (hubs) in each network are shown, as well as the links between them. Larger and reddish cells have a higher number of connections.

window model, is presented in Figure 10(b). This figure depicts 2% of the most connected nodes of the network and 3% of the network links, showing that for deep earthquakes networks the hubs do not concentrate a large number of connections between each other. This result implies that deep seismic events worldwide have no obvious correlation, making the understanding of their correlations more difficult than for the shallow ones.

CONCLUSIONS

In this work, the assortativity of networks of worldwide and synthetic earthquakes was studied in order to better characterize correlation properties and understand the spread of information in the system of earthquakes. Concerning the global events, we analyzed shallow and deep earthquakes separately. From these datasets, we built networks of epicenters by using two methodologies: the successive model and the time window model of connections. For the shallow earthquakes, the networks of both models of linking nodes (sites) presented assortative mixing, with the difference that the network constructed with the time window model was found to be much more assortative. These results were similar to those obtained for seismic event networks from California and Japan (regions with a predominance of shallow earthquakes).

The assortative mixing was also found for the network created using a catalog produced by an improved version of the computational model proposed by Olami, Feder and Christensen, which shows agreement between real data for shallow events and synthetic data catalogs. On the other hand, the deep earthquake networks, for both successive and time window models, presented no correlation between the degree of the nodes.

We have also tested our networks under two different constraints, taking only earthquakes with magnitude larger than or equal to 5.0 and excluding the events with "fixed depths" of 10 km from our data. The results found were consistent with those without the constraints. These tests had the objective to check if our results (using the entire data) were biased by the magnitude threshold 4.5 or by the "fixed depths".

Our results suggest that shallow and deep earthquakes have different temporal and spatial correlation properties. While we have positive degree correlations for shallow earthquakes, these correlations seem not to exist for deep earthquakes. Because of this, the shallow earthquake networks tend to link high-degree regions (regions with large earthquakes) with other high-degree areas, making it more difficult to change the seismological behavior of the earthquake networks even if a specific region stops having earthquakes for a period of time. Moreover, the results seem to indicate that, for shallow earthquakes, mainshocks may induce mainshocks in other areas, even if these areas are not close to each other. Another interesting feature revealed by our results from shallow earthquakes is a kind of "attracting dynamics". This feature causes the hubs to create several other hubs close to them; however, it does not prevent them from being connected to others further away. On the other hand, for deep earthquakes, our network analysis indicates that they connect randomly, i.e., with no specific preference.

An additional consequence of our findings is the validation of the OFC model to describe the degree correlation of shallow earthquake networks. It implies one more verification of the applicability of the OFC model. This step forward may help to study statistical correlations in actual earthquakes, given that the model presented similar results to those for natural earthquakes.

The correlation properties found in our analyses concern both temporal and spatial relations since we use time sequences to establish links between different spatial regions of the globe. It is known that earthquakes can induce others thousands of kilometers away from their epicenters (O'Malley et al., 2018), and our findings reinforce these long-range interactions in the worldwide seismicity. Here, these properties appear naturally using only complex network theory, showing how powerful this theory is to study correlated systems, such as earthquakes.

The present study shows that more than just degree distributions must be investigated when analyzing an earthquake network. Still, it is also necessary to observe how the network nodes are related. In this way, we have shown that the presence of mixing patterns in earthquake networks has a profound effect on the network's topological properties as it affects the detailed wiring of links among nodes.

For future works, we intend to study the earthquake clusters and communities performed from deep and shallow earthquakes.

ACKNOWLEDGMENTS

The authors would like to thank the Brazilian agencies FAPERJ and CNPq, for funding and scholarships.

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Received on November 7, 2021 / Accepted on April 7, 2022