

## ROBUST FILTERING OF GHOST AND PEG-LEG FROM SEA FLOOR SEISMIC DATA

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**ABSTRACT.** The processing of compressional waves from the acoustic components of ocean bottom seismic data is performed by decomposing the compressional wavefield into its upgoing and downgoing components, treating the first order multiples, composed by the overlap between the receiver ghost and peg-leg. The separation of these wavefields is achieved through the adaptive summation of the hydrophone and geophone components, usually in a least square sense. This method of separation is known as PZ summation, because it involves an operation between pressure and vertical particle velocity measurements. However, due to the difference in response of the pressure and velocity sensors, the premises assumed on the least square summation can be violated, degrading the results. To overcome these difficulties, a more robust method can be achieved using the  $L_1$  norm criterion for the adaptive sum. A comparison was made between the results obtained with the Iterative Reweighted Least Squares and the Wiener-Levinson filters. The robustness of the  $L_1$  norm sum was demonstrated by applications on the PZ summation of Ocean Bottom Cable data from the Jubarte area, in the Campos Basin, Brazil, showing improvements, especially when the multiples are present in the estimation window used to derive the filter coefficients.

**Keywords:** PZ summation; Wiener-Levinson; IRLS; ghost; peg-leg

### INTRODUCTION

The multi-component seafloor seismic has been used as a successful technique to improve the reservoir monitoring. This type of acquisition can increase the fold under production facility obstacles, as well as widen the azimuthal illumination and measure the converted shear wavefield. However, the processing flow for Ocean Bottom Cable (OBC) and Ocean Bottom Nodes (OBN) data differs from the one for conventional streamer data mainly due to the difference between source and receiver depths which provides an asymmetric raypath.

A method to process the hydrophone and geophone data from multi-component seismic survey was presented by [Amundsen and Reitan \(1995\)](#) and [Soubaras \(1996\)](#). Later, [Beresford and Janex \(1996\)](#), [Schalkwijk et al. \(2003\)](#), [Muijs et al. \(2007\)](#), [Wang and Grion \(2008\)](#), [Edme and Singh \(2009\)](#) and [Hugonnet et al. \(2011\)](#) addressed the theme of the ocean bottom seismic (OBS) data processing, explaining methods to separate the wavefield components under the receiver domain as well as their application in a real-world situation.

This paper concentrates on the acoustic wavefield immediately above the water bottom surface, under a unidimensional approach. Hugonnet et al. (2011) summarized the aspects of the hydrophone and geophone processing, based on Soubaras (1996), and called it as PZ summation (pressure with vertical particle velocity), generalizing it to the three-dimensional notation, in the least square sense. However, Guitton and Vershuur (2004) explain that, under non-gaussian distribution, when the multiple energy is present in the considered time window, the least square method provides poor adaptive sum to multiple attenuation, once that it is highly affected by spurious measures and, therefore, they suggest the use of the  $L_1$  norm.

We propose to apply a linear convolutional adapting filter based on the  $L_1$  norm instead of the least square filter in the formulation of the PZ summation to accomplish a unidimensional robust acoustic decomposition, which separates the upgoing and downgoing wave fields, for treating the receiver ghost and providing inputs to address the peg-leg multiple in the surveyed seafloor PP seismic data. The IRLS (Iterative Reweighted Least Squares) algorithm (Scales And Gersztenkorn, 1988) was used to obtain the filter coefficients and was applied to the PZ summation on OBC Seismic data from the Jubarte area, in the north of the Campos Basin, on the Espírito Santo coast, Brazil. A short explanation about the fundamentals of the least square and  $L_1$  norm filtering is showed. We also show the summarized formulation of the unidimensional PZ summation and the modification proposed in the adaptive sums.

**METHODS**

**The measured wavefields**

In conventional streamer acquisition, the receiver is commonly located few meters under the free-air surface, so the receiver side ghost notches contaminate only the low and the high frequency bands on the acquired data, affecting the data resolution. In ocean bottom acquisition, the noticeable depth of the receiver, related to free-air surface, provides ghost notch interference on the useful central band of the seismic data. The ghost arrives at the receiver in the downward direction; however, there is a synchronic event measured that arrives in the upward direction at the receiver, which is called peg-leg (Soubaras, 1996; Hugonnet et al., 2011). The overlap between them provides the water column delayed multiple event, measured in a sea-floor seismic survey. In terms of primary waves, to rightly process them, it is required to separate the wavefield into its upgoing and downgoing components from the hydrophone and the vertical geophone component receivers.

We can see in Figures 1 and 2 some events measured in a sea-floor seismic survey, with the depth of  $z$ , represented by a source  $S$ ,  $x$  away from the hydrophone  $H$  and the vertical geophone  $G$  at the same receiver location, in a bidimensional case. In Figure 1, the simplified upgoing wavefield component is showed, exposing the peg-leg trace set by the dashed rays from water bottom and the deep reflector. In the simplified downgoing wavefield (Figure 2), the ghost trace set is

represented by the dashed rays from water bottom and the deep reflector.

We can approximate this situation to a unidimensional case, showed in Figure 3. The letters  $D$  and  $U$  respectively represents the downward and upward arriving wavefields, while the subscripted  $d, wb, g, pg$  and  $p$  are, respectively, the direct wave, the water-bottom primary reflection, the ghost, the peg-leg and the deep primary reflections. Thus, Figure 3 illustrates the immediately above the water-bottom acoustic decomposition, where the subscript  $p$  refers to the possibility of expanding the approximation if we consider the acoustic media. This simplification can be applied to deep reflectors because the small source-receiver offset per target depth ratio allows a unidimensional treatment. Figures 1, 2 and 3 highlight the ghost and the peg-leg events by the dashed lines and vectors because they are considered as noises that compound the first order multiples in the sea-floor seismic data, when we follow the upgoing wavefield processing strategy, on which this search was focused. For the downgoing wavefield processing strategy, the ghost is separated into the downgoing wavefield and it is considered as a signal to accomplish the called mirror migration (Pacal et al., 2015).

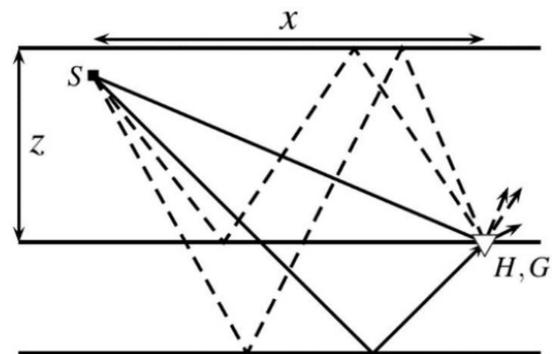


Figure 1: Upgoing wavefield.

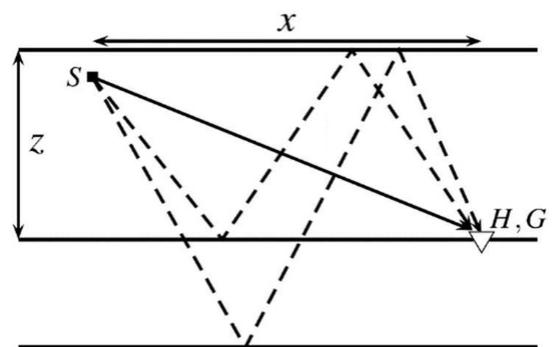


Figure 2: Down going wavefield.

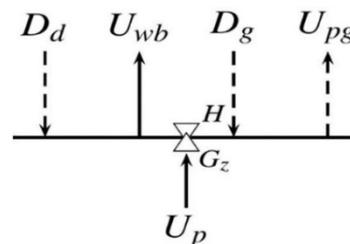


Figure 3: Unidimensional acoustic decomposition to sea-floor seismic data

## The pressure with vertical particle velocity summation

To treat the receiver ghost and peg-leg in ocean bottom multi-component seismic data, following the ongoing processing guideline, [Hugonnet et al. \(2011\)](#) summarized the formulation of PZ summation based on [Soubaras \(1996\)](#) by setting

$$U = H + f_0 * G \quad (1)$$

$$D = H - f_0 * G \quad (2)$$

where  $U$  and  $D$  are respectively the upgoing and downgoing wavefields;  $H$  and  $G$  are the hydrophone and geophone data components; and  $f_0 = \mathit{argmin} \|H' - f * G'\|_2^2$  is the least square filter. The elements  $H'$  and  $G'$  are defined in equations (4) and (5). The filter with the regularizing whitenoise  $\lambda$  ([Yilmaz, 2001](#)) is given by

$$f_0 = [G'^T G' + \lambda I]^{-1} G'^T H', \quad (3)$$

which can be numerical solved by the Wiener-Levinson (W-L) algorithm ([Golub and Van Loan, 1996](#); [Rosa, 2018](#)).

The adaptive sum is applied due to differences between the hydrophone ([Figure 4](#)) and geophone ([Figure 5](#)) signals, related mainly to coupling and their scalar and vector measures which provide an orthogonal phase delay between them. In addition, because of the polarizing feature of the geophone, the notches from the receiver ghost appear in the amplitude spectra with opposite peaks and troughs on the hydrophone and geophone components ([Figure 6](#)). To show a qualitative presentation for the synthetic trace modeling, the ghost amplitude was not attenuated by the geometrical spreading. The ghost is detected at three times the time of the primary event in [Figures 4](#) and [5](#), so the peg-leg overlaps it with a magnitude equal to the square power ghost amplitude, amplifying the amplitude level

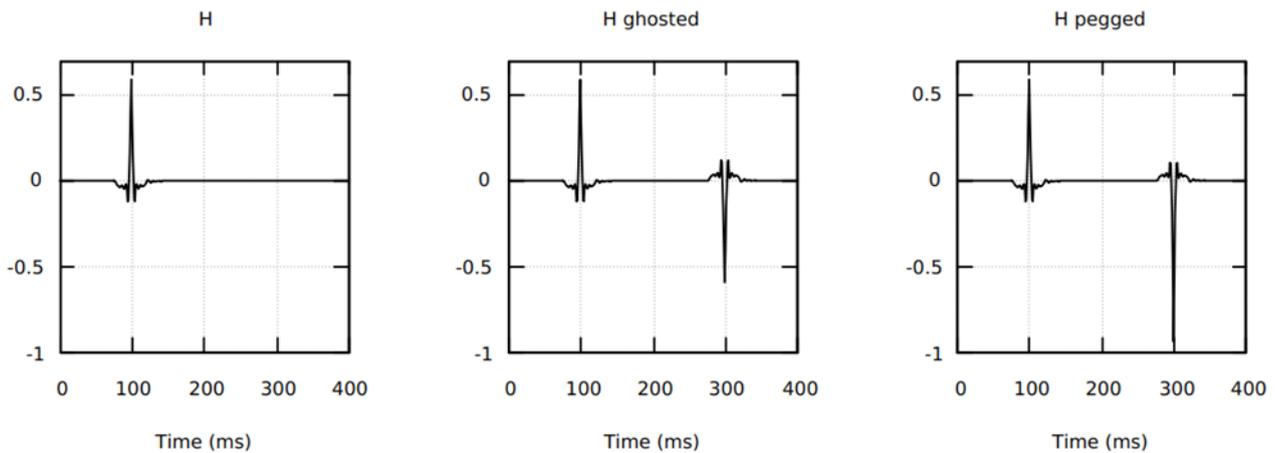


Figure 4: Hydrophone trace simulating. From left to right: primary event, ghost and peg leg overlapping the ghost.

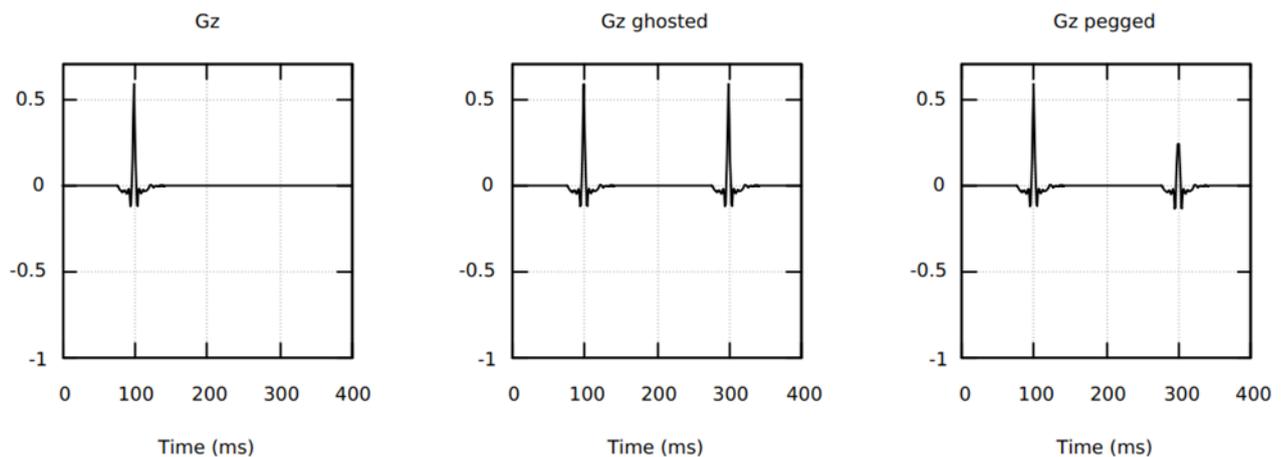


Figure 5: Vertical geophone trace simulating. From left to right: primary event, ghost and peg-leg overlapping the ghost.

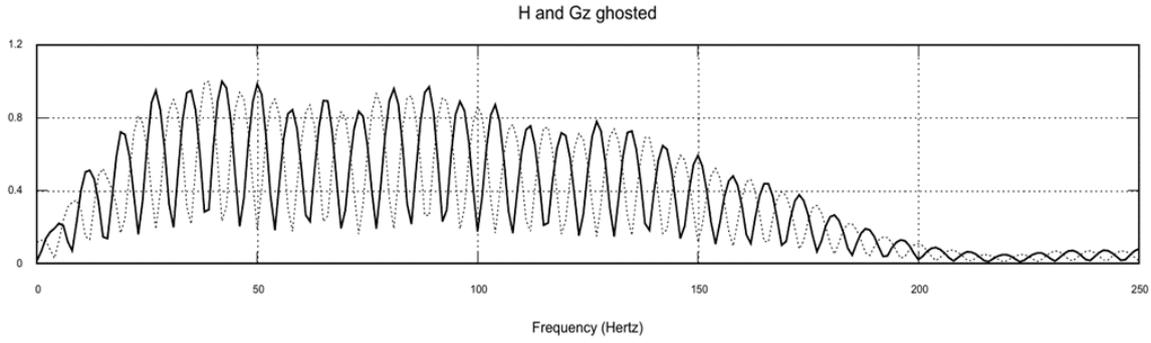


Figure 6: Normalized amplitude spectra for the synthetic hydrophone represented by the continuous black line, and the geophone by the dashed line, contaminated by its receiver ghost. Note the alternating notches.

To compensate this effect, [Soubaras \(1996\)](#) proposed the crossghosting operation preceding the filter calculation, making

$$H' = g_0 * H, \tag{4}$$

$$G' = h_0 * G, \tag{5}$$

where  $h_0$  and  $g_0$  are respectively the deterministic first order hydrophone and geophone ghost operators, which were built in time domain considering the normal incidence angle ( $\theta = 0$ ) and time delay as  $\Delta t = 2z \cos \theta / v$  ([Figure 7](#)). The reflectivity  $r$  from free air surface which is about -1, the water column velocity  $v$  by around 1500 meters per second and the geometric spreading coefficient  $E$ , by around 0.98 ([Rosa, 2018](#)), provide us

$$h_0 = \delta_t - rE\delta_{t-\Delta t} \tag{6}$$

$$g_0 = \delta_t + rE\delta_{t-\Delta t}, \tag{7}$$

where  $\delta_t$  is the delta function.

To attenuation of the peg-leg, we have the adaptive sum

$$P = U - p_0 * D, \tag{8}$$

where  $P$  is the un-pegged data;  $U$  and  $D$  are respectively the upgoing and downgoing wavefields from deghosting; and

$$p_0 = [D^T D + \lambda I]^{-1} D^T U \tag{9}$$

is the least square predictive filter, once the downgoing is a delayed version of the upgoing wavefield ([Soubaras, 1996](#); [Edme and Singh, 2009](#); [Ang et al., 2010](#); [Hugonnet et al., 2011](#)).

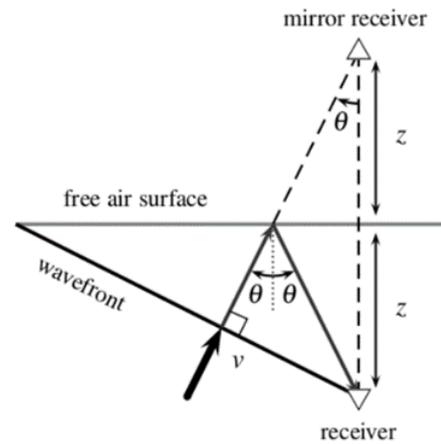


Figure 7: Associated geometry for the receiver ghost delay. Adapted from [Rosa \(2018\)](#).

### The Robust PZ summation

With the crossghosting operation and the IRLS method stated ([Appendix](#)), we propose a small modification on formulation of the acoustic decomposition, substituting the least square filter by the  $L_1$  norm-based filter obtained by IRLS. For this, we have that

$$U = H + f_k * G \tag{10}$$

$$D = H - f_k * G, \tag{11}$$

where

$$f_k = \operatorname{argmin} \| |H' - f * G'| \|_1 \tag{12}$$

is now the filter given by

$$f_k = [G'^T A G' + \mu B]^{-1} G'^T A H', \tag{13}$$

which was solved by Cholesky factorization (Golub and Van Loan, 1996) because the system matrix is positive definite (Li et al., 2016). In this case, we have the modeling deviation as

$$\Delta = H' - f_{k-1} * G'. \quad (14)$$

The parameter  $\mu$  refers to the system matrix regularization. Note that  $\mathbf{A}$  and  $\mathbf{B}$  are the IRLS method system matrices described in the Appendix by equations A3 and A5, in which there is a parameter  $\varepsilon$  that controls the  $L_1$  norm approximation accuracy.

A similar formulation can be developed to the depegging, so that

$$P = U - p_k * D, \quad (15)$$

where the predictive filter

$$p_k = [D^T A D + \mu B]^{-1} D^T A U \quad (16)$$

However, in this case, the modeling deviation is

$$\Delta = U - p_{k-1} * D. \quad (17)$$

Note that  $\mathbf{A}$  and  $\mathbf{B}$  are now built from the new modeling deviation given by equation 17.

## Data preconditioning – Designature

The method was applied to the hydrophone and the accelerometer of the OBC data 0364\_4D\_JUBARTE\_PRM\_MONITOR\_01 from the Jubarte area in the north of the Campos Basin. A receiver line parallel and close to a shot line, to simulate a bidimensional survey and evaluate the method on the receiver stacked lines and the receiver gathers from the real data.

Before submitting the data to the robust PZ summation, a designature processing was applied, using a statistically derived wavelet estimated from the flattened water bottom from the receiver stacked line, to calculate the predictive filter to debubbling and an equivalent zero phase wavelet. After designature, the data were resampled to 4 milliseconds, the half of the original input sample rate, and submitted to the anti-alias filtering.

The hydrophone was low cut filtered to attenuate the anomalous low frequency band, and the accelerometer was integrated to simulate the geophone data, once the particle acceleration is the derivative of the particle velocity.

## Data processing – Deghosting

The deghosting process was divided into two steps: filter calculation and filter application on the receiver gather. During the first step, just one filter is calculated

for each receiver gather, using an estimation window on the near receiver stacked section of each component, which is detailed in Figure 8. Note that on Figure 8 there are some relative anomalous amplitude noises over the stacked vertical component. Therefore, the level of the amplitude for the hydrophone component is four orders of the magnitude greater than the level for the geophone component. Thus, the integrated accelerometer was gain calibrated to establish the numerical operations during the filter calculation. The sum ran on the hydrophone and the vertical geophone receiver stacked sections to evaluate the filter quality. After the calculation, the filter was applied on the whole receiver gather traces, and the trace-to-trace sum ran before the stack. The filter quality was evaluated on both gathers and stacked sections. We used two sizes for the estimating window on the receiver stack sections, a long, between 0.0 and 10.0 seconds, and a short, between 1.06 and 2.48 seconds, and then we evaluated the results.

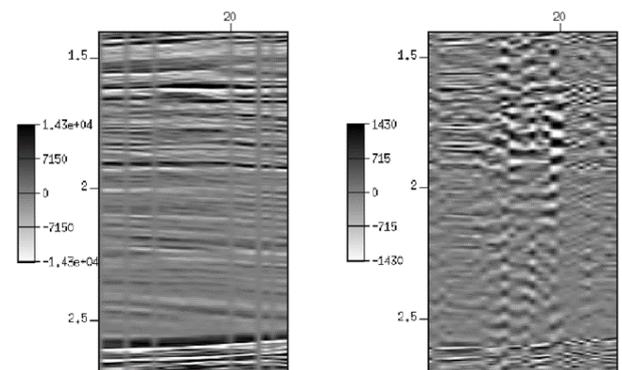


Figure 8: Details on long window deghosting receiver stack lines. From left to right, it can be seen the input designatured filtered hydrophone and the input designatured integrated vertical accelerometer.

A trace-to-trace short predictive filtering was accomplished to do the attenuation of the peg-leg, using the IRLS short window outputs from the deghost process as the input data. The upgoing and downgoing receiver gathers were submitted to the adaptive sum, providing the unpegged data. After the filtering, the receiver gathers were stacked providing the receiver stack lines.

## RESULTS

### Deghosting – long window

The crossghosting was applied before the filter calculation due to the multiple events contained in the long estimation window. The multiple energy inserts spurious conditions which violate least squares premises. The results on stacked receiver lines, during the filter calculation step, verified the robustness of the IRLS filtering over the Wiener-Levinson filtering.

Though the results observed on stacked receiver lines, when the method was applied on the receiver gathers (Figure 9), artifacts contaminate the resulting wavefield receiver gathers due to the unidimensional assumption used to obtain the ghost operator. If we consider the real incidence angle regarding the offset trace in the receiver gather and we calculate a bi-dimensional receiver ghost operator, these artifacts can disappear. This artifact is pointed by arrow number 4 in Figure 9, while the real event is pointed by arrow number 3. Arrow number 1 refers to the primary reflection in the upgoing and downgoing gathers. As we considered a normal incidence on the horizontal water bottom reflector, the receiver ghost operator was built only for this unidimensional case, obtaining good results for the near offset traces in the receiver gathers and coinciding with the interception zone (pointed by arrow number 2 in Figure 9) between the ghost operator artifact and the real ghost event, which provides the new receiver stack lines (Figure 10) verifying the robustness of the method proposed.

### Deghosting – short window

Because the estimating short window is between the water bottom and the first order multiple, the multiple energy does not generate the spurious conditions, and the notches do not appear in the component amplitude spectra. Thus, the crossghosting step was excluded from the filter calculation. The results show us similar quality in the gathers and the stacked receiver lines with negligible differences, that enhance the raw amplitude spectra

in useful frequency band in the IRLS upgoing gathers and reduce the raw amplitude level in high frequency band from the quality control window (between the water bottom reflection and the first order multiple event) in the IRLS downgoing gathers, in comparison with the Wiener-Levinson results. Due to the exclusion of the multiples from inside the short estimation window, the ghost operator artifact does not appear in the resulting receiver gathers.

### Depegging

The depegging proceeding was applied with short window IRLS upgoing and downgoing wavefields as inputs. The results of the applied IRLS predictive filter show us similar quality in the gathers and in their corresponding receiver stacked lines, when compared to the Wiener-Levinson predictive filter results. Figure 11 shows the data evolution on the receiver stack lines for the upgoing wavefield processing strategy, from the pressure and the particle velocity, which is represented by the integrated accelerometer receiver stack section, to the short windowed IRLS deghosted and the IRLS depegged receiver stack sections. Note that the first output resulted with similar quality as the hydrophone section, although, in this case, just the peg-leg remains in the deghosted upgoing section. The resulting IRLS depegged section obtained intermediate quality between the hydrophone and the integrated accelerometer sections, showing a low level randomly noised section, where it is noticed that the peg-leg became inconspicuous.

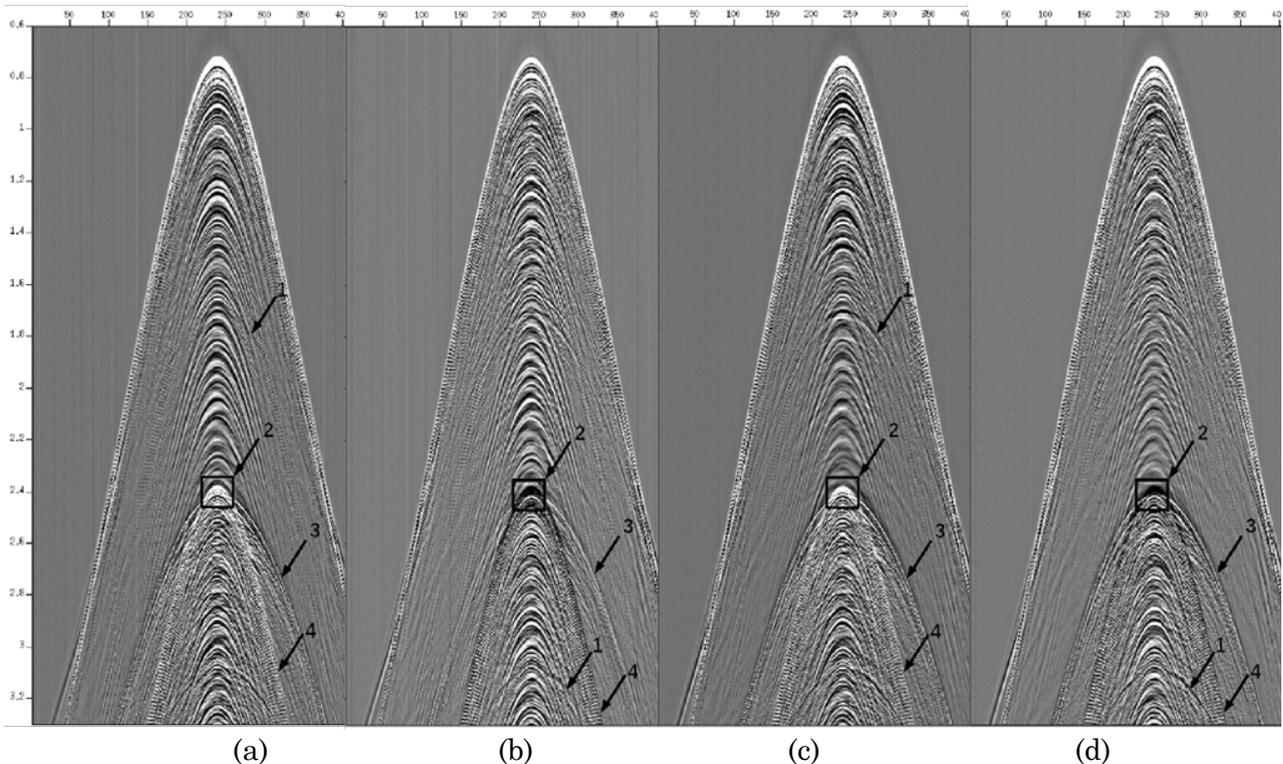


Figure 9: Long window deghosting receiver gathers: (a) Wiener-Levinson upgoing; (b) Wiener-Levinson downgoing; (c) IRLS upgoing; (d) IRLS downgoing. The vertical scale is in seconds.

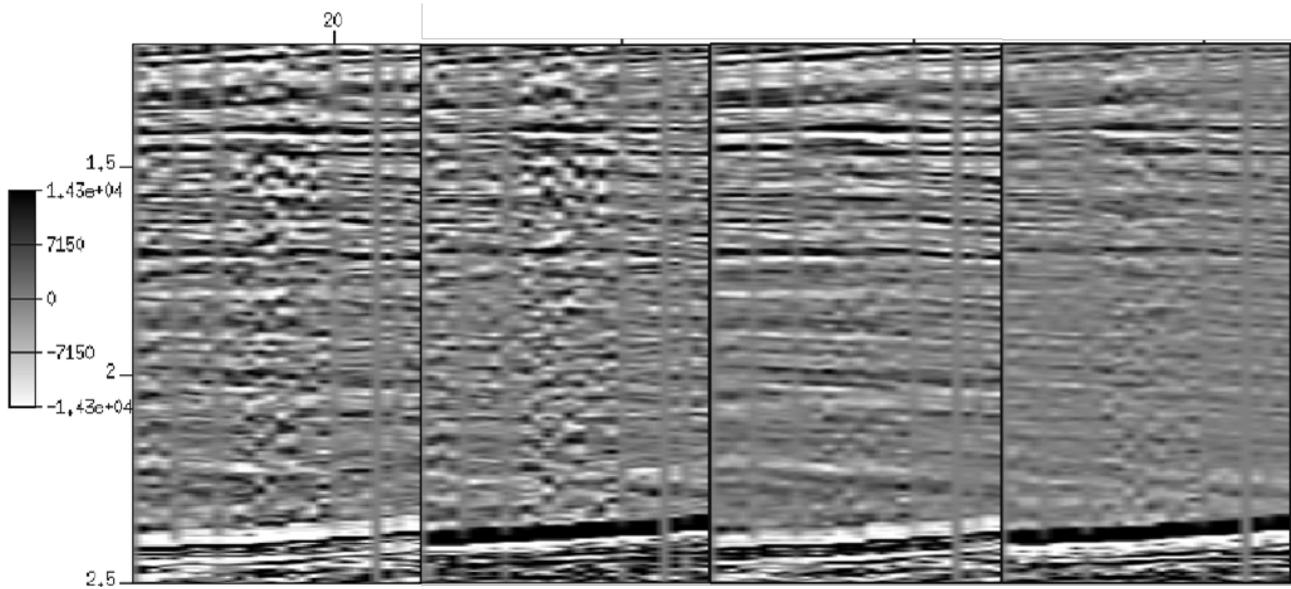


Figure 10: Details on long window deghosting receiver stack lines: from left to right, the Wiener-Levinson upgoing; the Wiener-Levinson downgoing; the IRLS upgoing; and the IRLS downgoing.

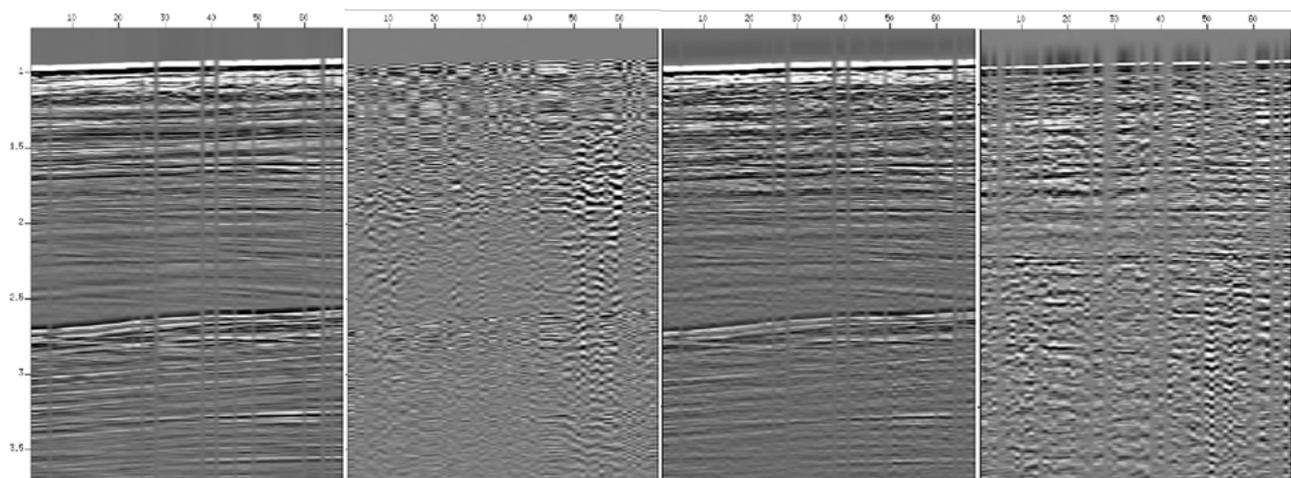


Figure 11: Data evolution on receiver stack lines. From left to right: Hydrophone input; Vertical Integrated Accelerometer input; IRLS upgoing output and IRLS depegged output. The vertical scale is in second.

## CONCLUSIONS

This work showed that filters based on the  $L_1$  norm are a good alternative to the conventional Wiener-Levinson filters based on the  $L_2$  norm normally adopted in the treatment of ghost and peg-leg multiples present in seafloor seismic data. This was evident in the case where the multiples are present in the data window used to derive the coefficients of such filters.

The implementation of the  $L_1$  Filter requires the solution of a non-linear system of equations. For this, we present a mathematical procedure based on the IRLS method. This method has a higher computational cost, since it requires  $n^3/3$  plus  $2n^2$  float point operations to solve triangular systems using Cholesky in each iteration while the Wiener-Levinson filter requires only  $n^2$  float point

operations.

We recommend a previous processing preceding the PZ summation, as the designation and some noise attenuation. To perform the acoustic decomposition for deghosting, we must evaluate the size of the water column, to select the correct estimation window. When the estimation window contains multiple reflections, i.e., superposition between receiver ghost and peg leg, we recommend the application of the crossghosting operations before the filter calculation. Thus, on shallow water bottom seismic data, it is difficult to determine an estimation window free of multiples; and on deep water bottom, the unidimensional solution for the ghost operator generates artifacts on receiver gather outputs.

The introduced method can be expanded to multi-dimensional case, if the correct incidence angle is calculated for each offset trace in the gather, so that the deterministic ghost operator can be correctly calculated, avoiding artifacts in outputs, as well as the filtering operation running to the matrix or volumetric filter (multi-dimensional convolution).

## APPENDIX

### The Iterative Reweighted Least Square method

The IRLS method is based on asymptotic approximation  $g(x)$  of the  $L_1$  norm curve  $f(x)$ :

$$P = U \cdot p_k * D, \quad (A1)$$

where  $\varepsilon$  is a small value (Figure A1).

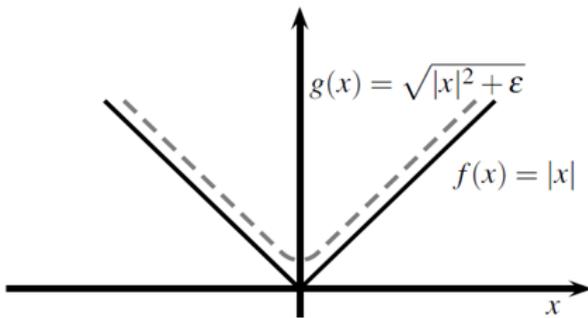


Figure A1: Asymptotic approximation  $g(x)$  which allows the differentiation of the  $L_1$  norm curve  $f(x)$ .

Developing the derivative based on this approximation (equation A1), we found the nonlinear system of the IRLS method as

$$[M^T A M + \mu B] f_k = M^T A X \quad (A2)$$

which can be iteratively solved. Note that

$$A = \text{diag}(1/\sqrt{\Delta^2 + \varepsilon}), \quad (A3)$$

$$\Delta = |X - M f_k|, \quad (A4)$$

and

$$B = \text{diag}\left[1/(f_{k-1}^2 + \varepsilon)^{\frac{1}{2}}\right] \quad (A5)$$

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